# PAST, PRESENT AND FUTURE

# OF

# EARTHQUAKE ANALYSIS OF STRUCTURES

*By Ed Wilson*

***September 24, 2014***

***SEAONC Lecture #2***

# *Terminology in nonlinear analysis Which does not have a unique definition*

1. *Equal Displacement Rule*
2. *Pushover Analysis*
3. *Equivalent Linear Damping*
4. *Equivalent Static Analysis*
5. *Nonlinear Spectrum Analysis*
6. *Onerous Response History Analysis*

# ***Summary of Lecture Topics***

***Field measurements of frequencies and mode shapes***

***Exact Eigenvectors or Approximate Ritz Vectors***

***The Load Dependent Ritz Vectors – LDR Vectors***

***The Fast Nonlinear Analysis Method – FNA Method***

***Error Estimation – Conservation of Energy***

***Foundation – Structure Interaction***

***The Retrofit of the Richmond-San Rafael Bridge***

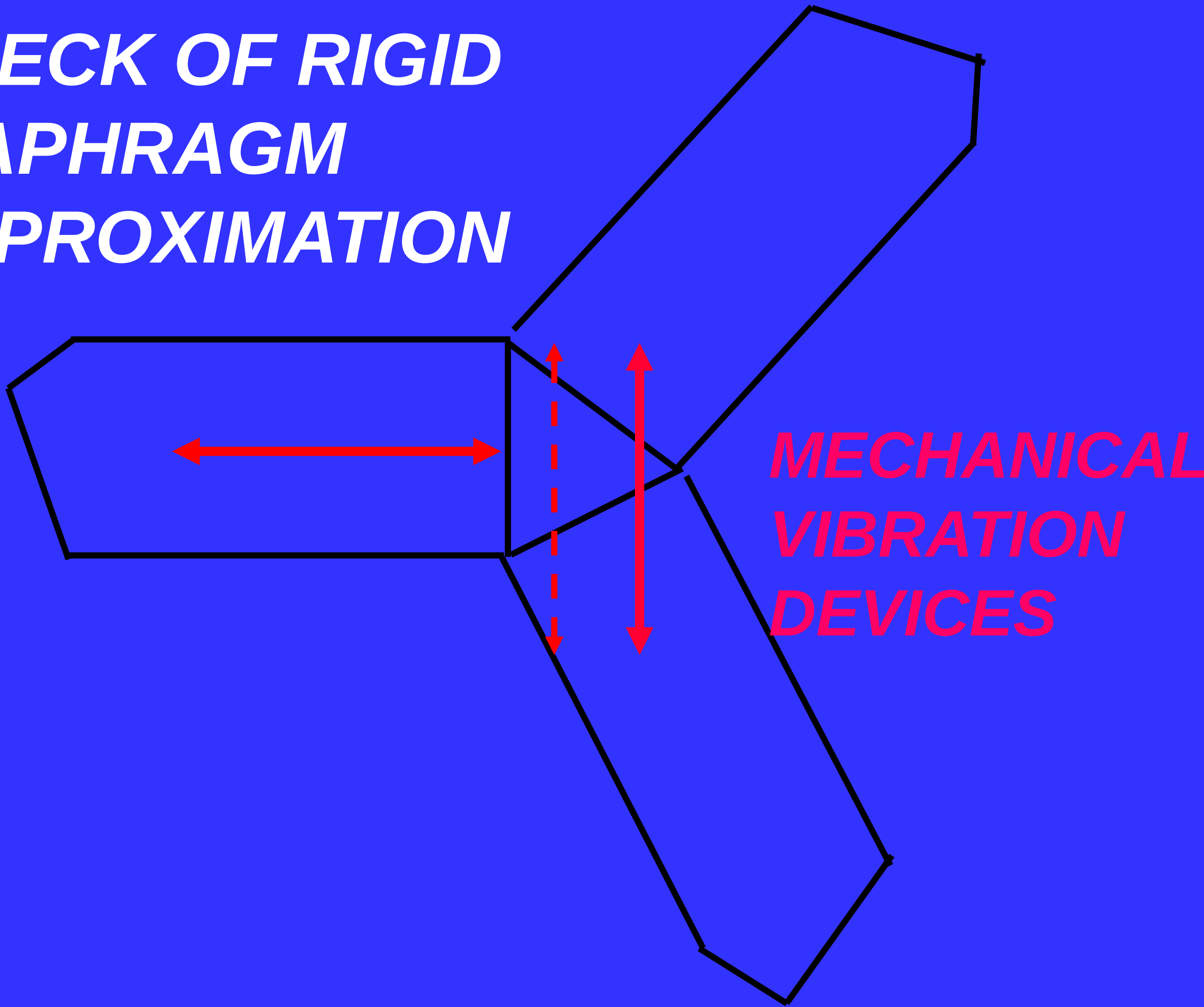
***Recommendations***

# *FIELD MEASUREMENTS REQUIRED TO VERIFY*

1. ***MODELING ASSUMPTIONS***
2. ***SOIL-STRUCTURE MODEL***
3. ***COMPUTER PROGRAM***

***Resulted in program modification***

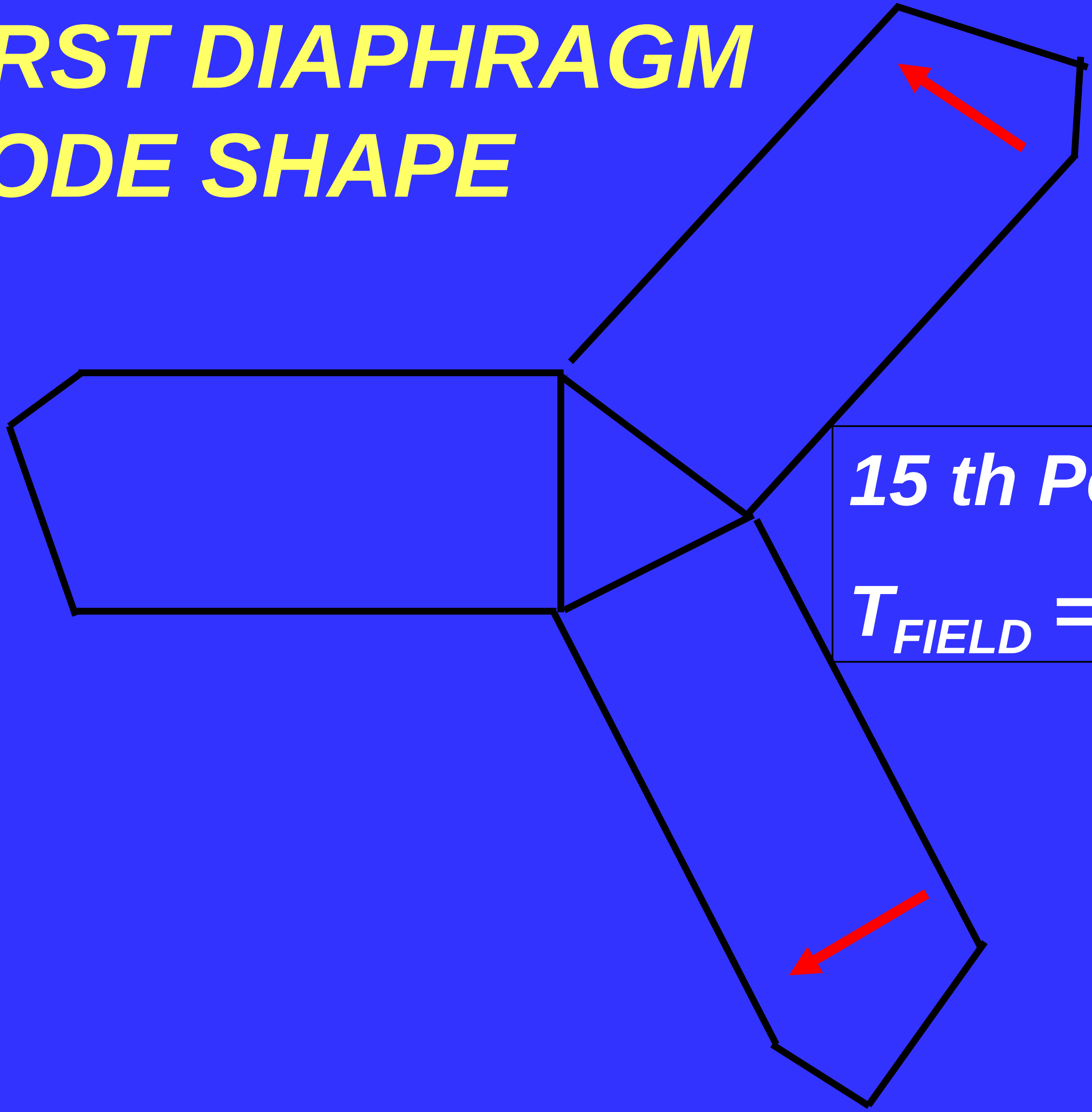
# CHECK OF RIGID DIAPHRAGM APPROXIMATION



# FIELD MEASUREMENTS OF PERIODS AND MODE SHAPES

| MODE | $T_{FIELD}$ | $T_{ANALYSIS}$ | Diff. - % |
|---|---|---|---|
| 1 | 1.77 Sec. | 1.78 Sec. | 0.5 |
| 2 | 1.69 | 1.68 | 0.6 |
| 3 | 1.68 | 1.68 | 0.0 |
| 4 | 0.60 | 0.61 | 0.9 |
| 5 | 0.60 | 0.61 | 0.9 |
| 6 | 0.59 | 0.59 | 0.8 |
| 7 | 0.32 | 0.32 | 0.2 |
| - | - | - | - |
| 11 | 0.23 | 0.32 | 2.3 |

# FIRST DIAPHRAGM MODE SHAPE

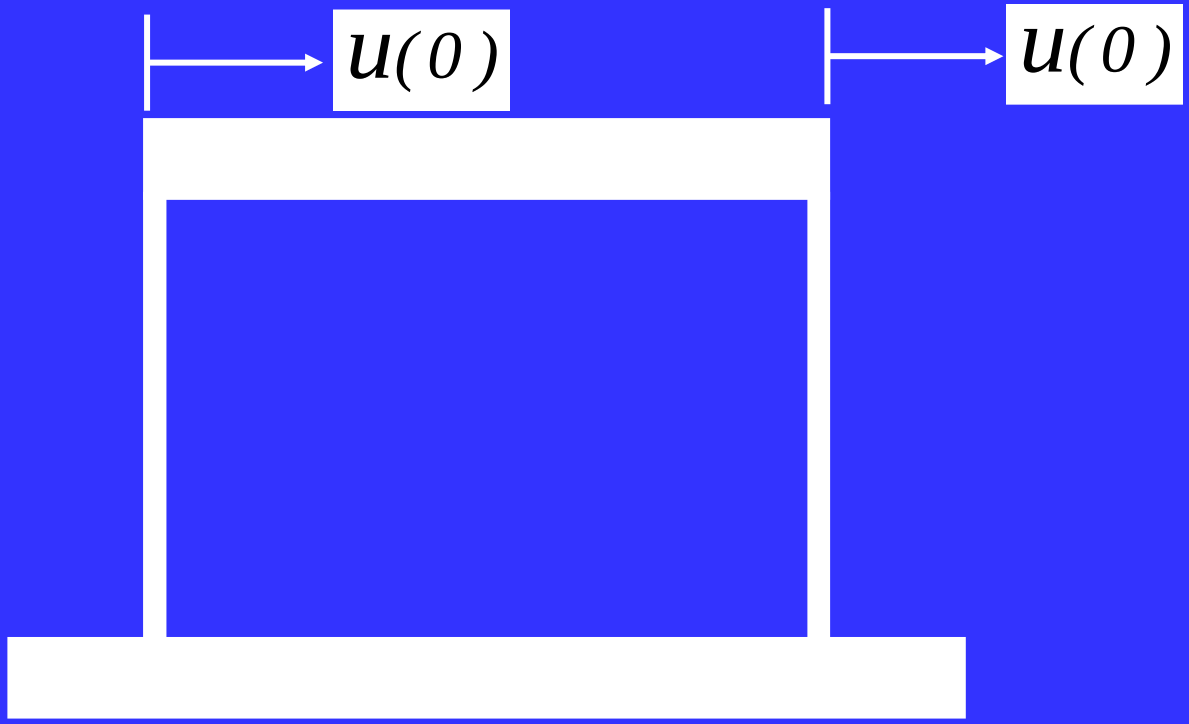


*15 th Period*

*$T_{FIELD}$ = 0.16 Sec.*

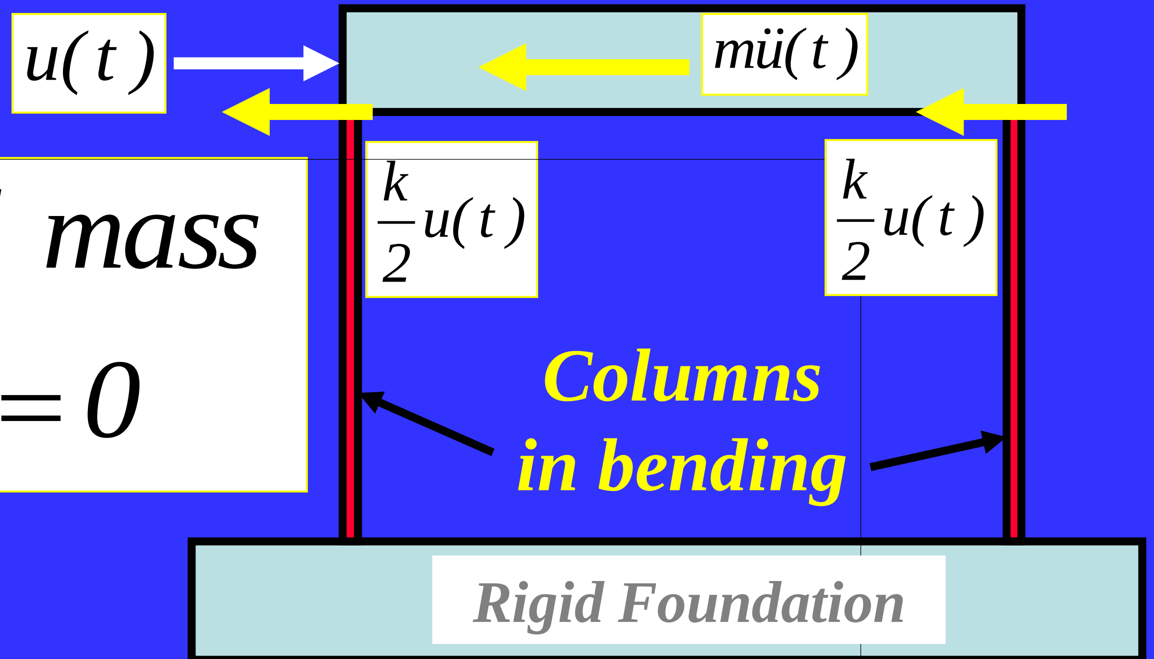
# Simple Example of Dynamic Response

# With no external load applied




Initial displacement $u(0)$

# Simple Example of Free Vibration




$$\text{Equilibrium of mass}$$

$$m\ddot{u}(t) + ku(t) = 0$$

*For a Static Initial Condition of* $u(0) = 1.0$

*The Solution of the Force Equilibrium Equation is* $u(t) = \cos(\omega t) \quad for\ t = 0\ to\ \infty$

***Check Solution:***

*Since* $\ddot{u}(t) = -\omega^2 \cos(\omega t) \quad for\ t = 0\ to\ \infty$

$$-\omega^2 m + k = 0 \qquad Where,\quad \omega = \sqrt{\frac{k}{m}}\ \ rad/sec \qquad Or,\ f = \frac{\omega}{2\pi}\ cps$$

# Physical Analysis of Free Vibration

***Displacement*** $u(t) = \cos(\omega t)$ ***Velocity*** $\dot{u}(t) = -\omega \sin(\omega t)$

***Strain Energy*** $E_s(t) = \frac{1}{2} k u(t)^2 = \frac{1}{2} k \cos^2(\omega t)$

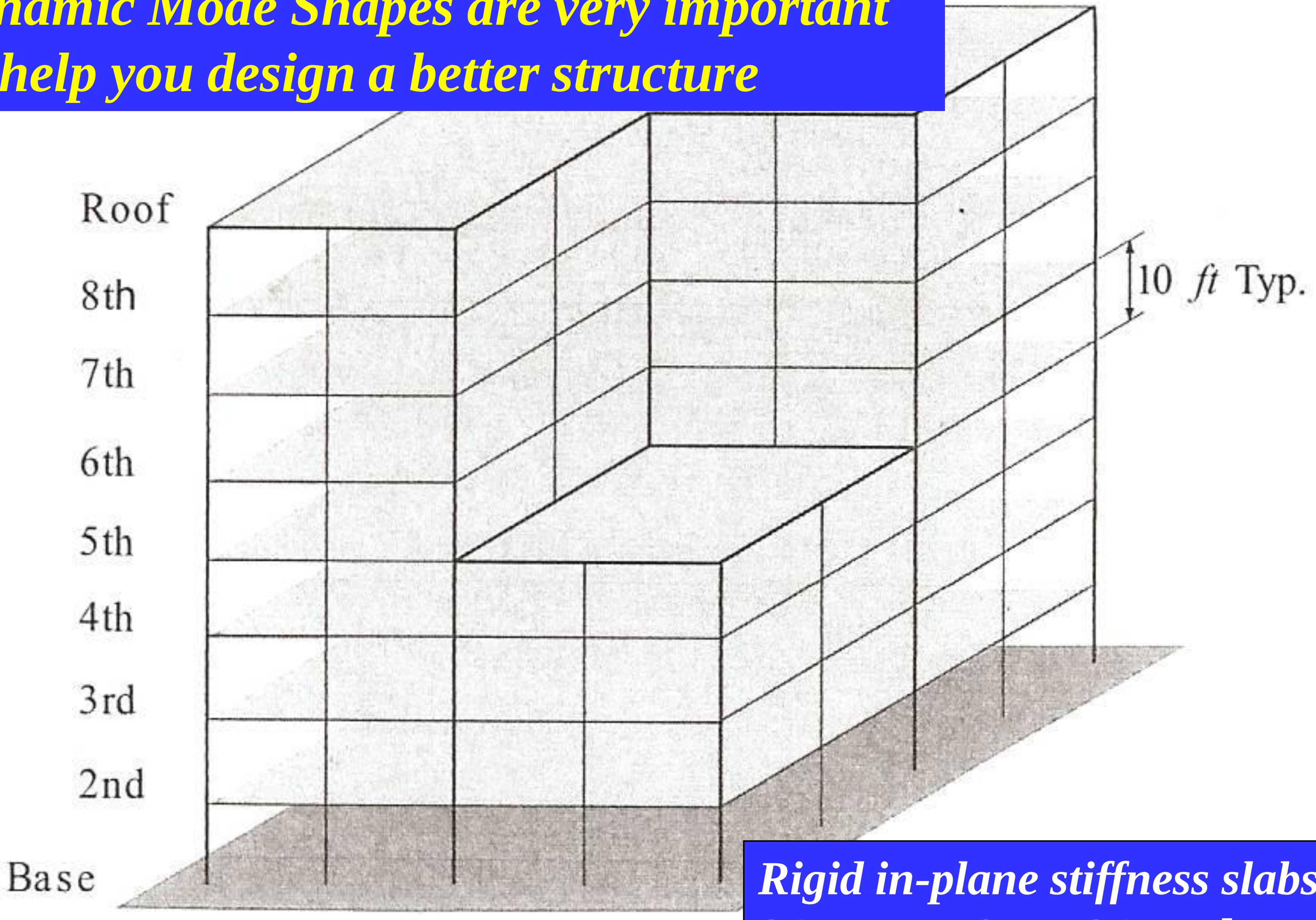
***Kinetic Energy*** $E_k(t) = \frac{1}{2} m \dot{u}(t)^2 = \frac{1}{2} m \omega^2 \sin^2(\omega t) = \frac{1}{2} k \sin^2(\omega t)$

***Total Energy*** $E(t) = E_k(t) + E_s(t) = \frac{1}{2} k [\sin^2(\omega t) + \cos^2 \omega t] = \frac{1}{2} k$

***Without Energy Dissipation the System would Vibrate Forever***

## *Energy Pump*

**Dynamic Mode Shapes are very important**
**To help you design a better structure**




**Rigid in-plane stiffness slabs**
**24 Mass DOF - 24 Modes**

**Figure 17.1:** Example of Eight-Story Irregular Building

# *Modal Base Shears and Directions and Overturning Moments for each Mode*

| Mode | Period (*sec*) | Modal Base Shear Reactions | | | Modal Over-Turning Moments | | |
|---|---|---|---|---|---|---|---|
| | | $V_x$ (*kips*) | $V_y$ (*kips*) | Angle (*deg*) | $M_x$ (*kip-ft*) | $M_y$ (*kip-ft*) | $M_z$ (*kip-ft*) |
| 1 | 0.6315 | 0.781 | 0.624 | 38.64 | −37.3 | 46.6 | −18.9 |
| 2 | 0.6034 | −0.624 | 0.781 | −51.37 | −46.3 | −37.0 | 38.3 |
| *3* | *0.3501* | *0.785* | *0.620* | *38.30* | *−31.9* | *40.2* | *85.6* |
| 4 | 0.1144 | −0.753 | −0.658 | 41.12 | 12.0 | −13.7 | 7.2 |
| 5 | 0.1135 | 0.657 | −0.754 | −48.89 | 13.6 | 11.9 | −38.7 |
| *6* | *0.0706* | *0.989* | *0.147* | *8.43* | *−33.5* | *51.9* | *2,438.3* |
| 7 | 0.0394 | −0.191 | 0.982 | −79.01 | −10.4 | −2.0 | 29.4 |
| 8 | 0.0394 | −0.983 | −0.185 | 10.67 | 1.9 | −10.4 | 26.9 |
| *9* | *0.0242* | *0.848* | *0.530* | *32.01* | *−5.6* | *8.5* | *277.9* |

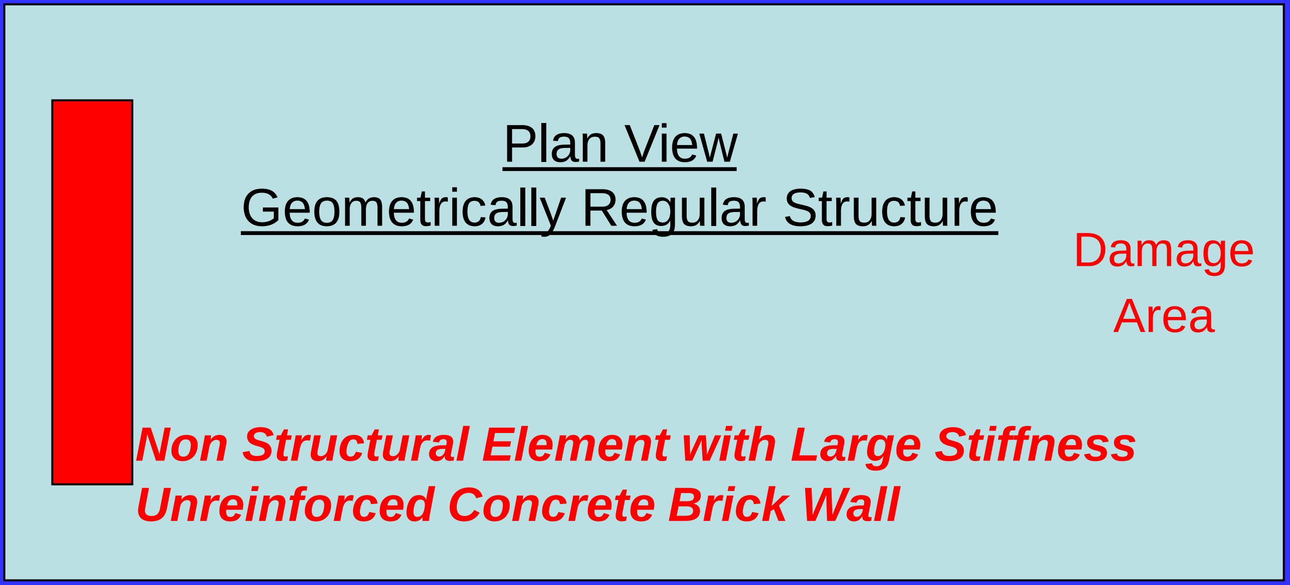
# *Regular and Irregular Structures*

***The current code defines an "irregular structure" as one that has a certain geometric shape or in which stiffness and mass discontinuities exist.***

***A far more rational definition is that a "regular structure" is one in has minimum coupling between the lateral displacements and the torsional rotations for the mode shapes associated with the lower frequencies of the system.***

***Therefore, if the model is modified and "tuned" by studying the three-dimensional mode shapes during the preliminary design phase, it may be possible to convert a "geometrically irregular" structure to a "dynamically regular" structure from an earthquake-resistant design standpoint.***

# *A Dynamic Irregular Structure*



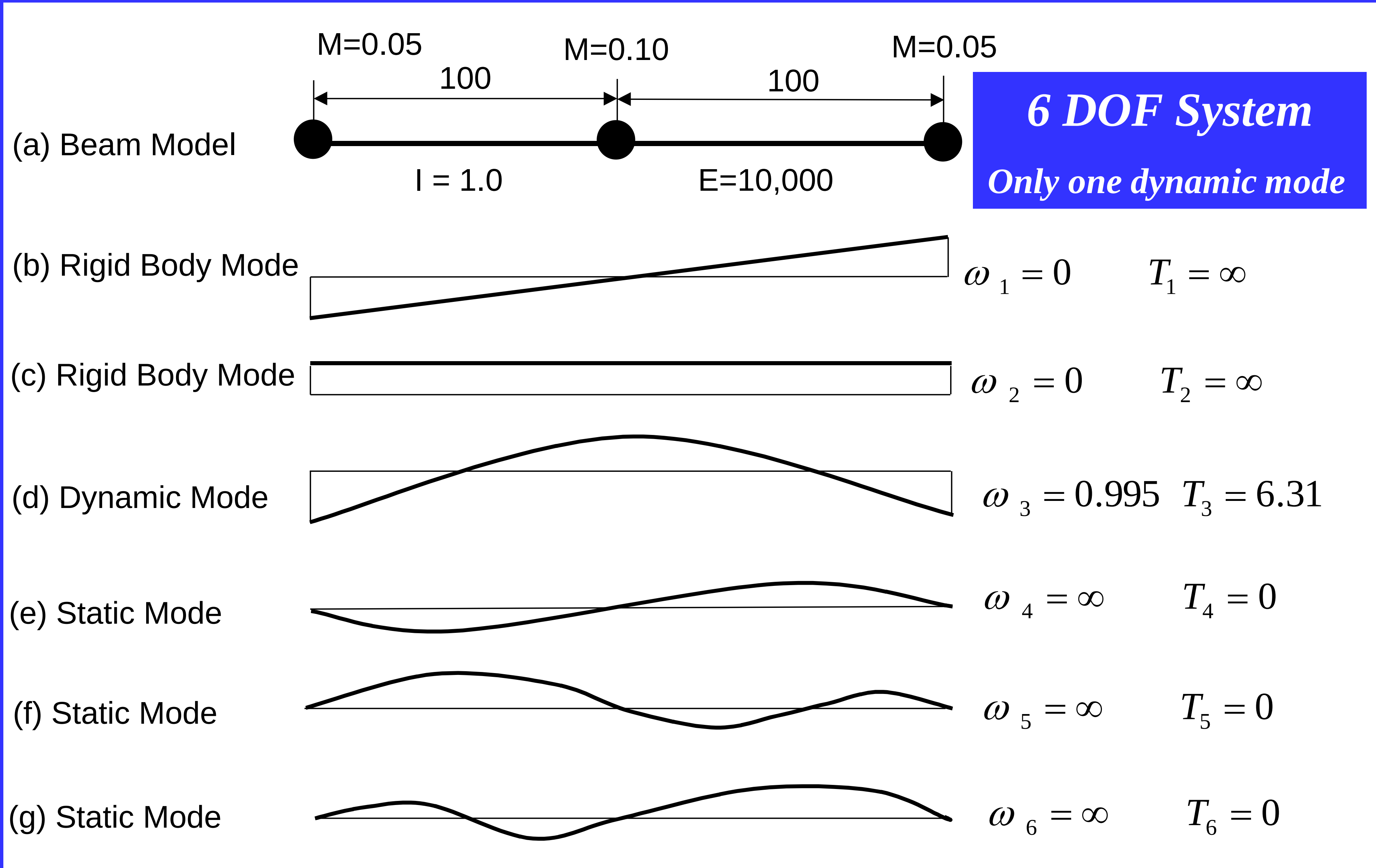
**Carl Gustav Jacob Jacobi** (1804 –1851)

***was a German who made fundamental contributions to classical mechanics, dynamics and astronomy. A crater on the Moon is named after him.***

***Jacobi first presented the method for the calculation of mode shapes and frequencies in 1846 (168 year ago).***

***After using the method for over 50 years, I have concluded it is the most robust numerical method for the calculation of mode shapes and frequencies.***

***It never fails to produce results***.




# *Example of the Jacobi Method for the Evaluation of Mode Shapes and Frequencies*

## *For the Earthquake Analysis of Structures the Load Dependent Ritz method produces more Accurate results than the uses of the Exact Eigenvectors*

*Fewer LDR mode shapes are required*

*Less computation time is required*

*Zero and infinite frequencies are calculated, or,*

*dynamic, static and rigid body modes are found*

*The LDR vectors can be used for nonlinear analysis*

# ***L**oad-**D**ependent **R**itz Vectors*

*LDR Vectors – 1980 – 2000*

*14.8 Page 157*

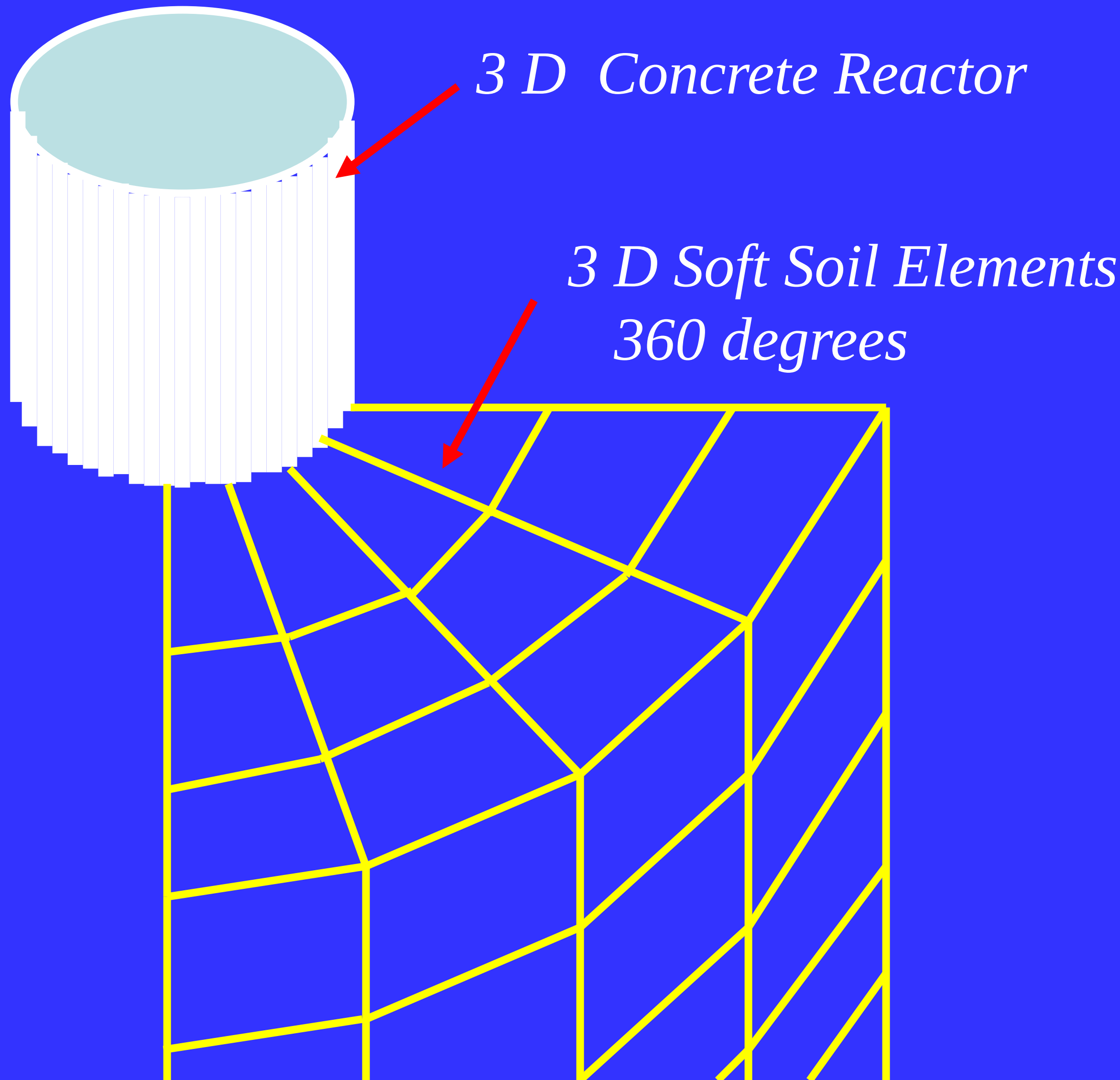
# *MOTAVATION – 3D Reactor on Soft Foundation*

***Dynamic Analysis - 1979 by Bechtel using SAP IV***

*200 Exact Eigenvalues were Calculated and all of the Modes were in the foundation – No Stresses in the Reactor.*

*The cost for One analysis on the CRAY Computer was*

***$10,000***



# Linear Dynamic Equilibrium Equation

$$M\ddot{u}(t)+C\dot{u}(t)+Ku(t)+=\sum_{j}^{L} f_j g_j(t)=F_0 G(t)$$

$First, solve\ for\ static\ displacements \quad u_0=K^{-1}F_0\approx V_0$

$where\ has\ K=_{;} LDL^T\ has\ been\ factored$

***The LDR Vectors are calculated by :***

$Solve\ by\ iteration\ \ i=1,2,........N\ blocks$

$u_i=K^{-1}MV_{i-1}\approx V_i$ *(this is an error estimation from previous block)*

$where\ \approx V_i \quad indicates\ all\ vectors\ are\ made\ stiffness\ and$

$mass\ orthogonal\ using\ the\ Jacobi\ Method\ for\ each\ step\ i$

# Generation of Load Dependent ritz Vectors

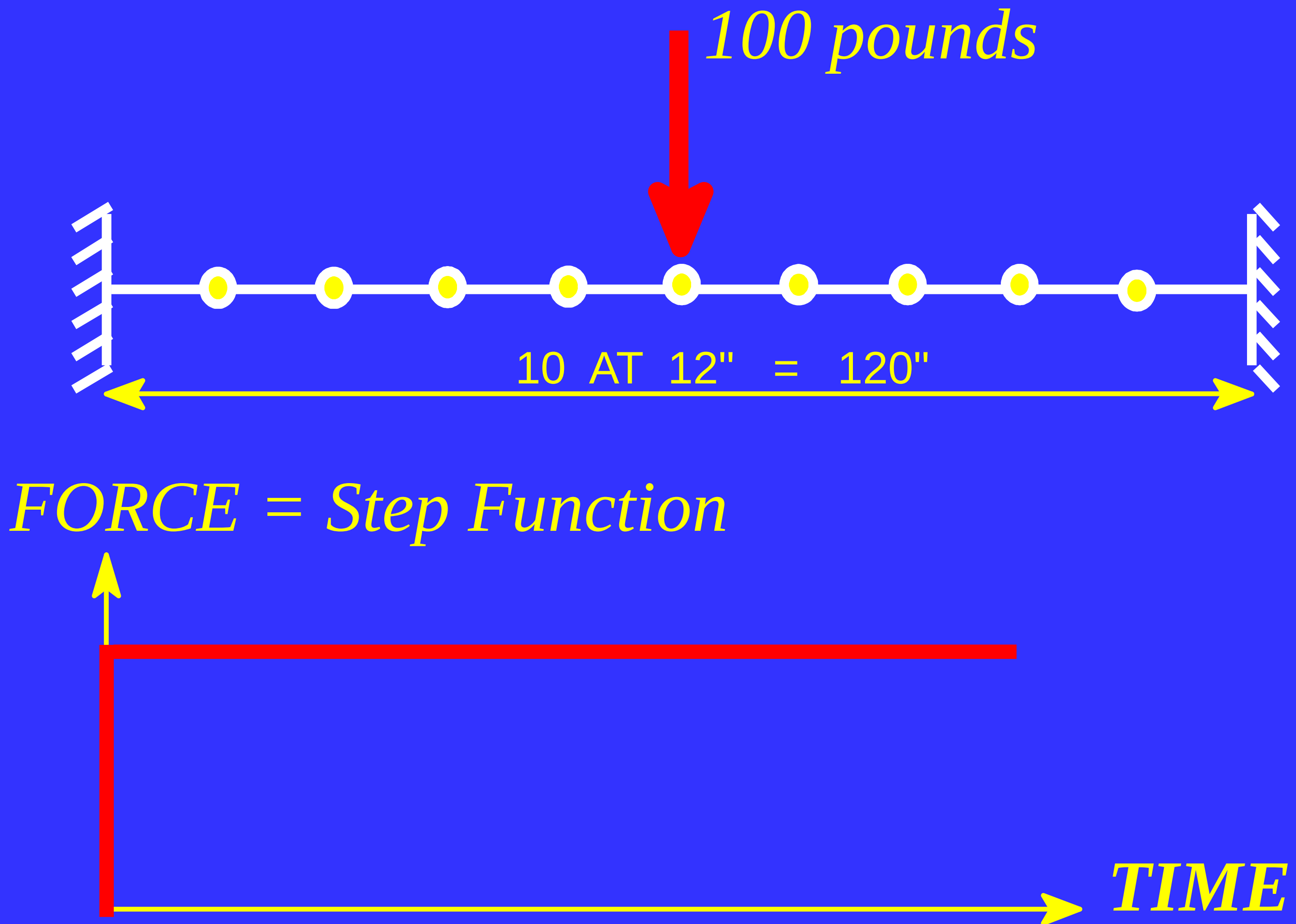
**1.** *Approximately Three Times Faster Than The Calculation Of Exact Eigenvectors*

*2. Results In Improved Accuracy Using A Smaller Number Of LDR Vectors*

*3. Computer Storage Requirements Reduced*

*4. Can Be Used For Nonlinear Analysis To Capture Local Static Response*

# *DYNAMIC RESPONSE OF BEAM*



# *MAXIMUM DISPLACEMENT*

| Number of Vectors | Eigen Vectors | | Load Dependent Vectors | |
|---|---|---|---|---|
| 1 | 0.004572 | (-2.41) | 0.004726 | (+0.88) |
| 2 | 0.004572 | (-2.41) | 0.004591 | ( -2.00) |
| 3 | 0.004664 | (-0.46) | 0.004689 | (+0.08) |
| 4 | 0.004664 | (-0.46) | 0.004685 | (+0.06) |
| 5 | 0.004681 | (-0.08) | 0.004685 | ( 0.00) |
| 7 | 0.004683 | (-0.04) | | |
| 9 | 0.004685 | (0.00) | | |

( Error in Percent)

# *MAXIMUM MOMENT*

| Number of Vectors | Eigen Vectors | | Load Dependent Vectors | |
|---|---|---|---|---|
| 1 | 4178 | ( - 22.8 %) | 5907 | ( + 9.2 ) |
| 2 | 4178 | ( - 22.8 ) | 5563 | ( + 2.8 ) |
| 3 | 4946 | ( - 8.5 ) | 5603 | ( + 3.5 ) |
| 4 | 4946 | ( - 8.5 ) | 5507 | ( + 1.8) |
| 5 | 5188 | ( - 4.1 ) | 5411 | ( 0.0 ) |
| 7 | 5304 | ( - .0 ) | | |
| 9 | 5411 | ( 0.0 ) | | |

*( Error in Percent )*

# LDR Vector Summary

*After Over 20 Years Experience Using the LDR Vector Algorithm*

*We Have Always Obtained More Accurate Displacements and Stresses*

*Compared to Using the Same Number of Exact Dynamic Eigenvectors.*

***SAP 2000 has Both Options***

# *The **F**ast **N**onlinear **A**nalysis Method*

*The FNA Method was Named in 1996*

*Designed for the Dynamic Analysis of Structures with a Limited Number of Predefined Nonlinear Elements*

# FAST NONLINEAR ANALYSIS

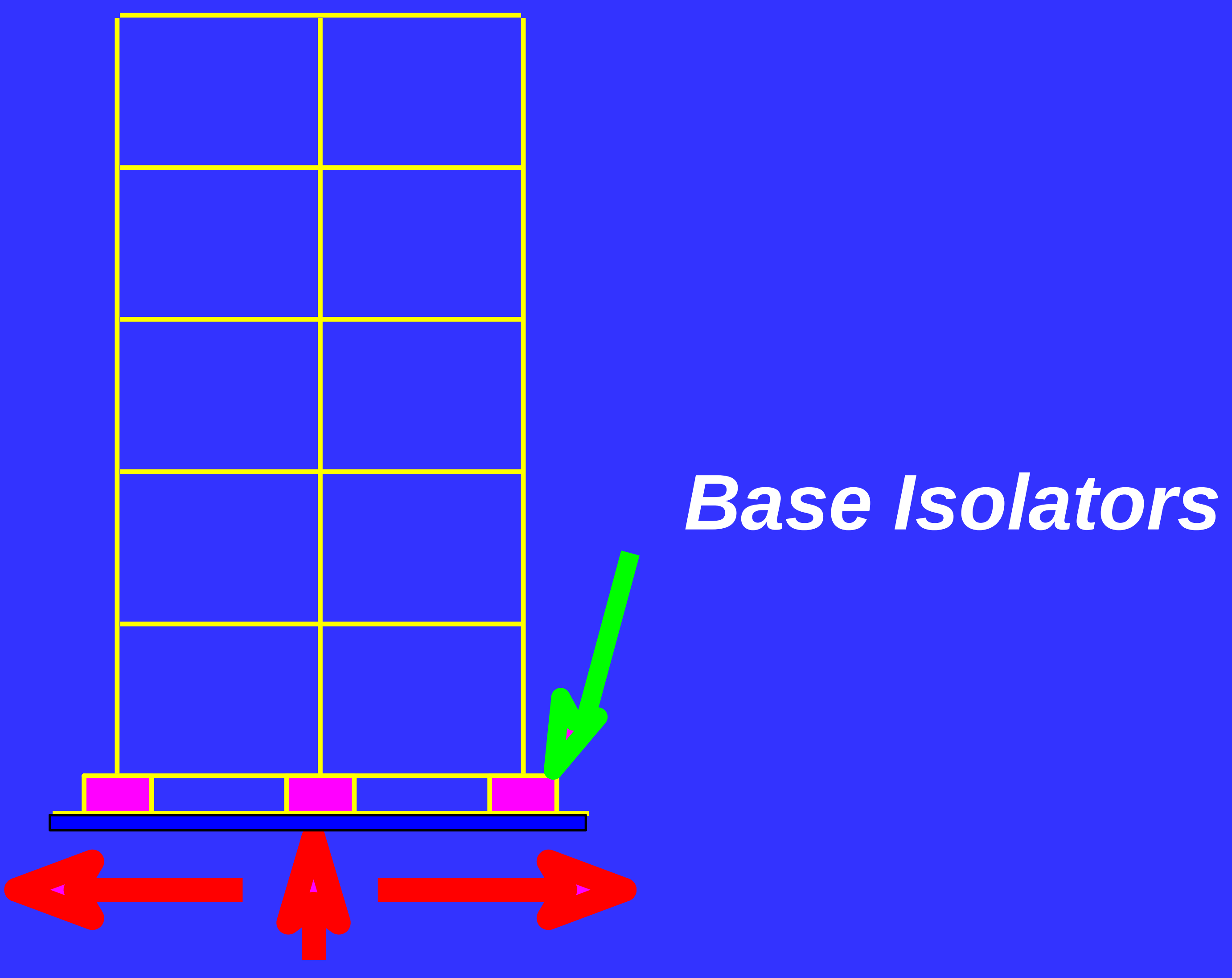
1. *EVALUATE* **LDR** *VECTORS WITH NONLINEAR ELEMENTS REMOVED AND DUMMY ELEMENTS ADDED FOR STABILITY*

2. *SOLVE ALL MODAL EQUATIONS WITH NONLINEAR FORCES ON THE RIGHT HAND SIDE*

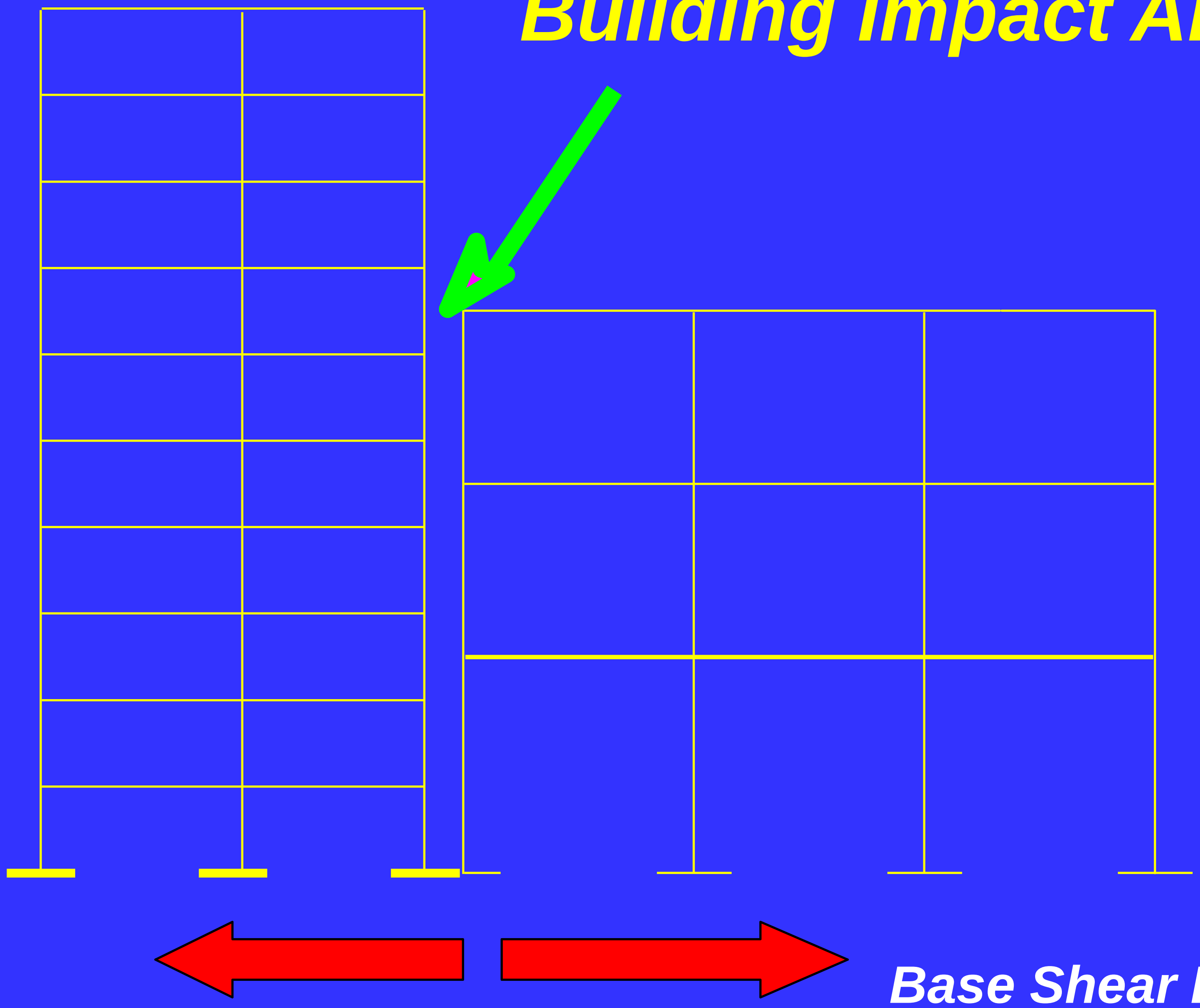
3. *USE EXACT INTEGRATION WITHIN EACH TIME STEP*

4. *FORCE AND ENERGY EQUILIBRIUM ARE STATISFIED AT EACH TIME STEP BY ITERATION*

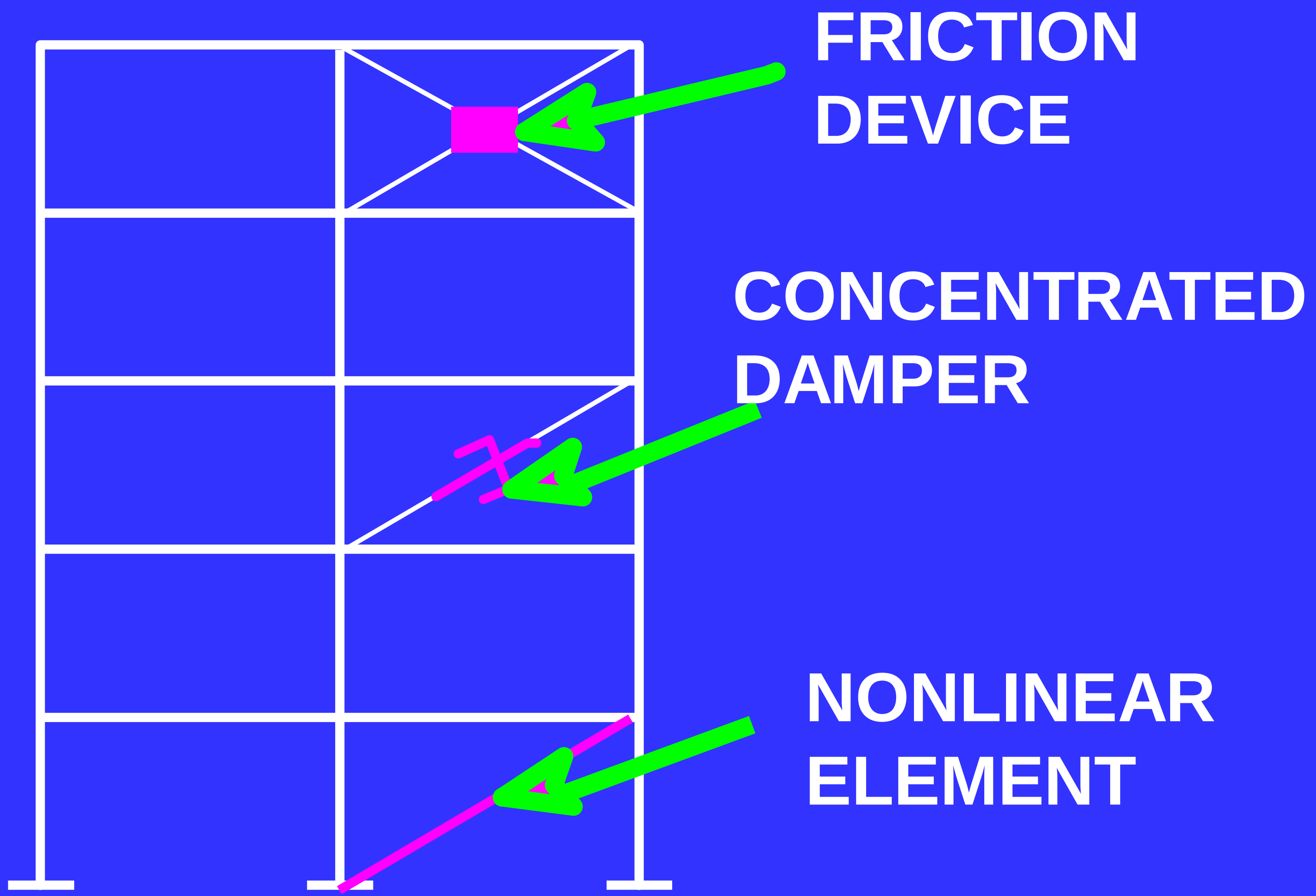
# BASE ISOLATION

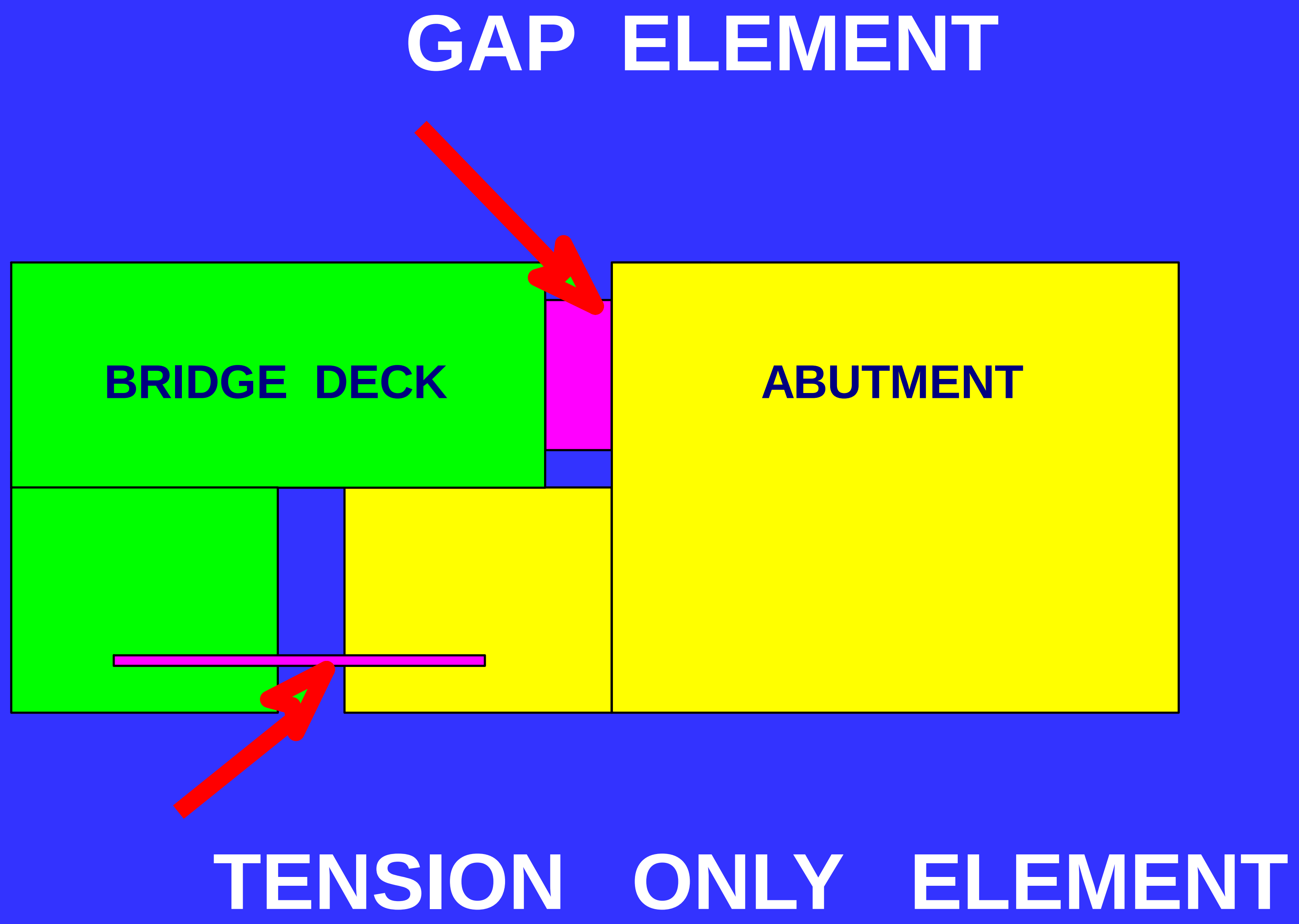


# Building Impact Analysis

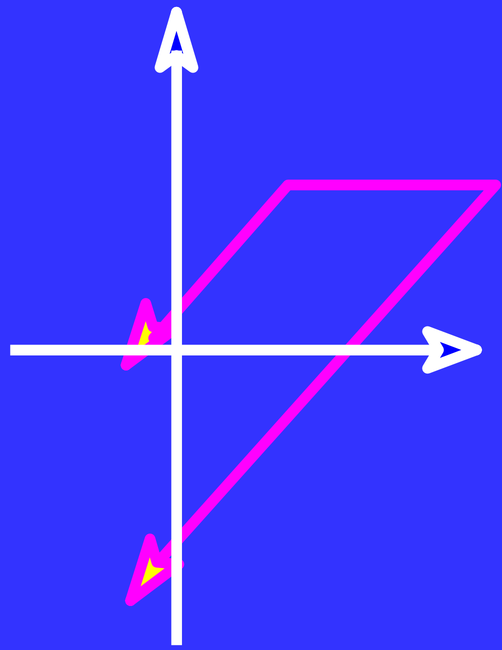


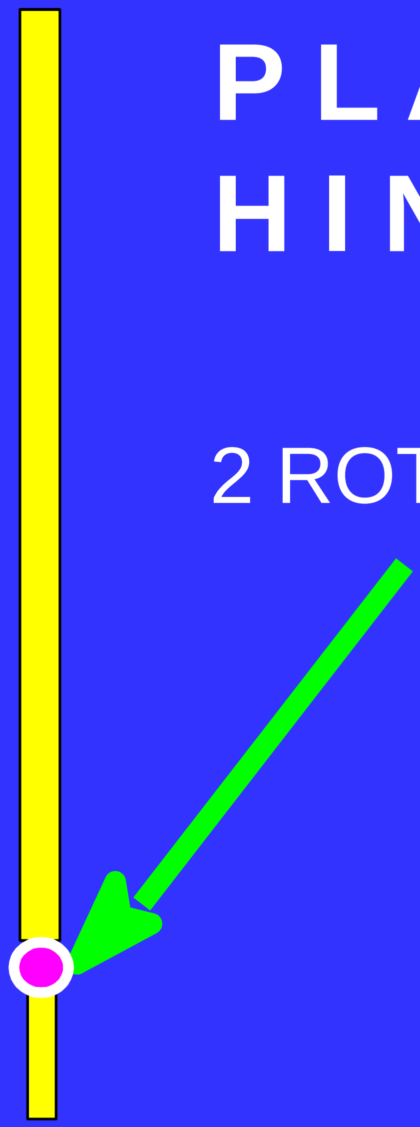
*Base Shear Equal Zero*


FRICTION
DEVICE
CONCENTRATED
DAMPER
NONLINEAR
ELEMENT


GAP ELEMENT
BRIDGE DECK
ABUTMENT
TENSION ONLY ELEMENT

# PLASTIC HINGES

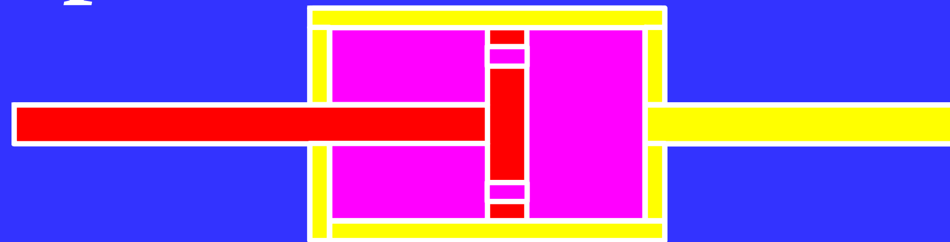




2 ROTATIONAL DOF

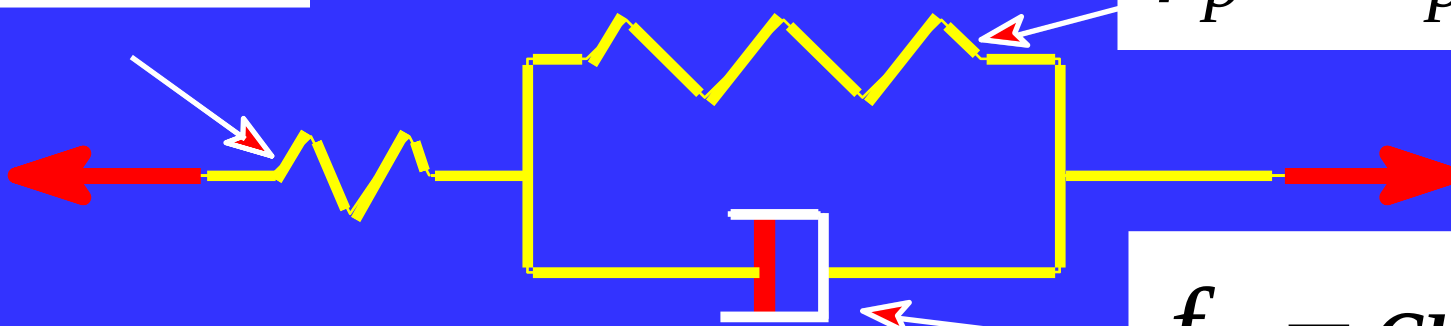
***DEGRADING STIFFNESS ?***

# Mechanical Dampers



$$f_s = k_s u_s$$

$$f_p = k_p u_p$$



$$f_d = c\dot{u}^n$$

# Mathematical Model

# *LINEAR VISCOUS DAMPING*

***Does not exist in normal structures and foundations***

***5 or 10 percent modal damping values are often used to justify energy dissipation due to nonlinear effects***

***If energy dissipation devices are used, then 1 percent modal damping should be used for the elastic part of the structure - CHECK ENERGY PLOTS***

# Comparison with Experimental Results

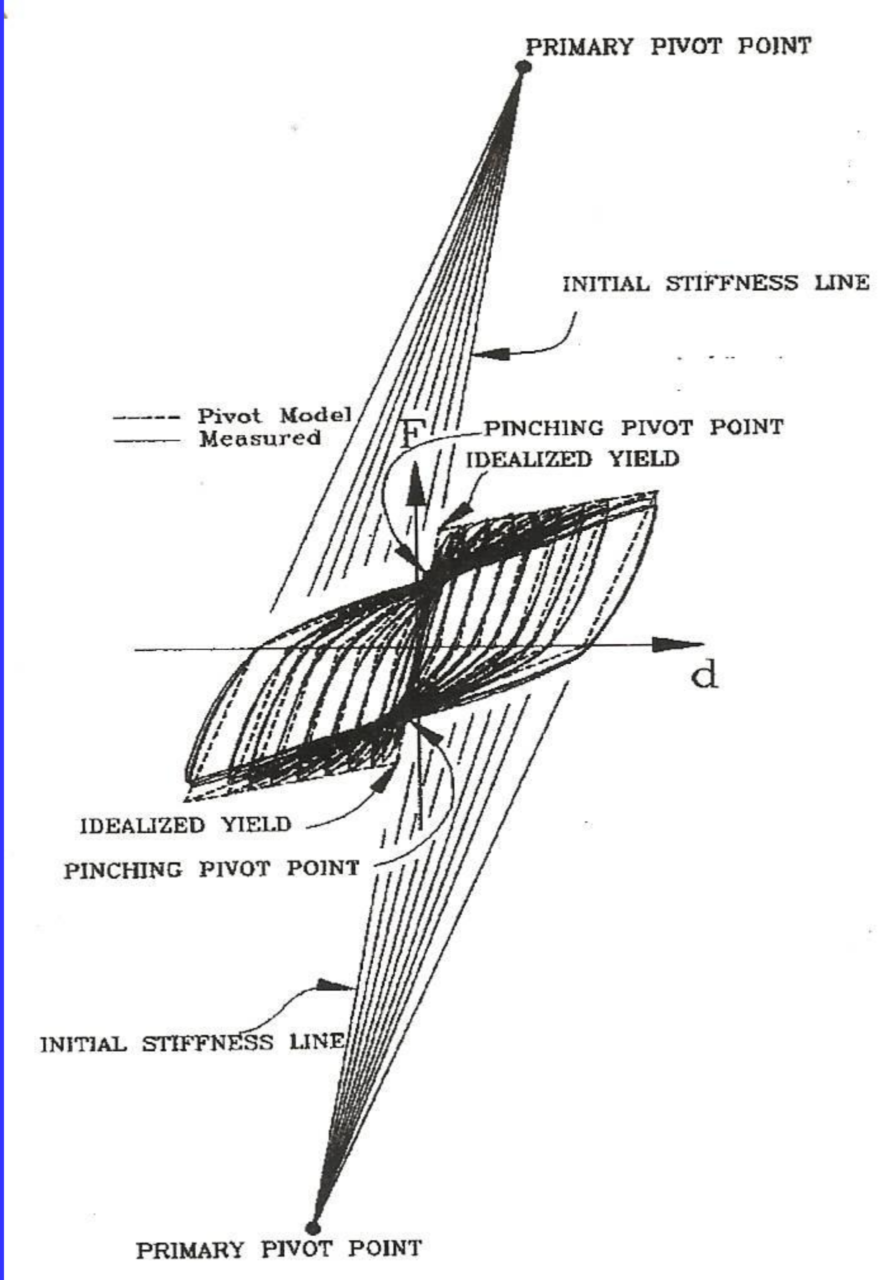
ACI STRUCTURAL JOURNAL

Title no. 95-S55

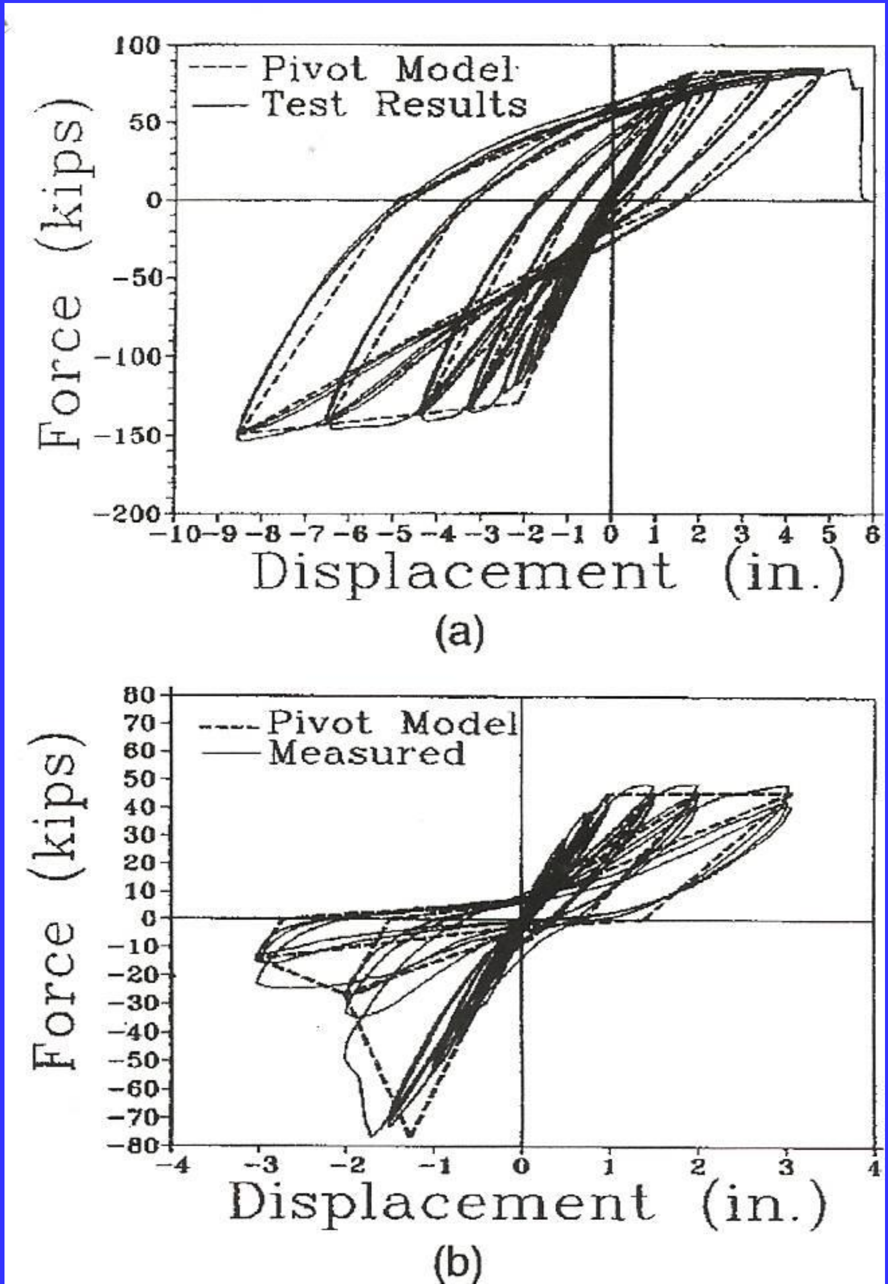
## Pivot Hysteresis Model for Reinforced Concrete Members

by Robert K. Dowell, Frieder Seible, and Edward L. Wilson

## *General Pivot Element*




## *Comparison with Test Results*



# ***Summary of the FNA Method***

***See Chapter 18 for details***

# Nonlinear Equilibrium Equations

$$M\ddot{u}(t)+C\dot{u}(t)+Ku(t)+F_N(t)=\sum_{j}^{L} f_j g_j(t)=F(t)$$

*After moving the nonlinear forces to the right hand side*

$$M\ddot{u}(t)+C\dot{u}(t)+Ku(t)=F(t)-F_N(t)$$

*In order to inprove the rate of convergene*

*(or to make the structure stable)*

*We can add an effective stiffness to both sides of the equation*

$$M\ddot{u}(t)+C\dot{u}(t)+\left[K+K_E\right]u(t)=F(t)+[K_E u(t)-F_N]$$

# Nonlinear Equilibrium Equations

$$M\ddot{u}(t) + C\dot{u}(t) + \overline{K}u(t) = \overline{F}(t) + K_E u(t) - F_N(t)$$

$$Let\ u(t) = \phi Y(t), \quad \dot{u} = \phi\dot{Y}(t) \quad and \quad \ddot{u}(t) = \phi\ddot{Y}(t)$$

*After premutiplication by* $\varphi^T$ *the modal equations are*

$$I\ddot{Y}(t) + c\dot{Y}(t) + \Omega^2 Y(t) = R(t) + R(t)_N$$

***Where*** $R(t)_N = \phi^T K_E u(t) - \phi^T F_N(t)$ ***must be solved by iteration at each time "t"***

*All Modal Equations are integrated at the same time since the modes are coupled by the nonlinear elements.*

***The deformations in the nonlinear elements can be calculated from the following displacement:***

$$d(t) = bu(t) = b\phi Y(t) = BY(t) \; where \; B = b\phi$$

$$Or, in \; iterative \; form, \quad d(t)^{(i)} = BY(t)^{(i-1)}$$

***Where the size of the B array is equal to the number of nonlinear deformations times the number of LDR vectors. This array is calculated only once prior to the start of mode integration. Also, the modal forces associated with the non-linear elements are calculated from***

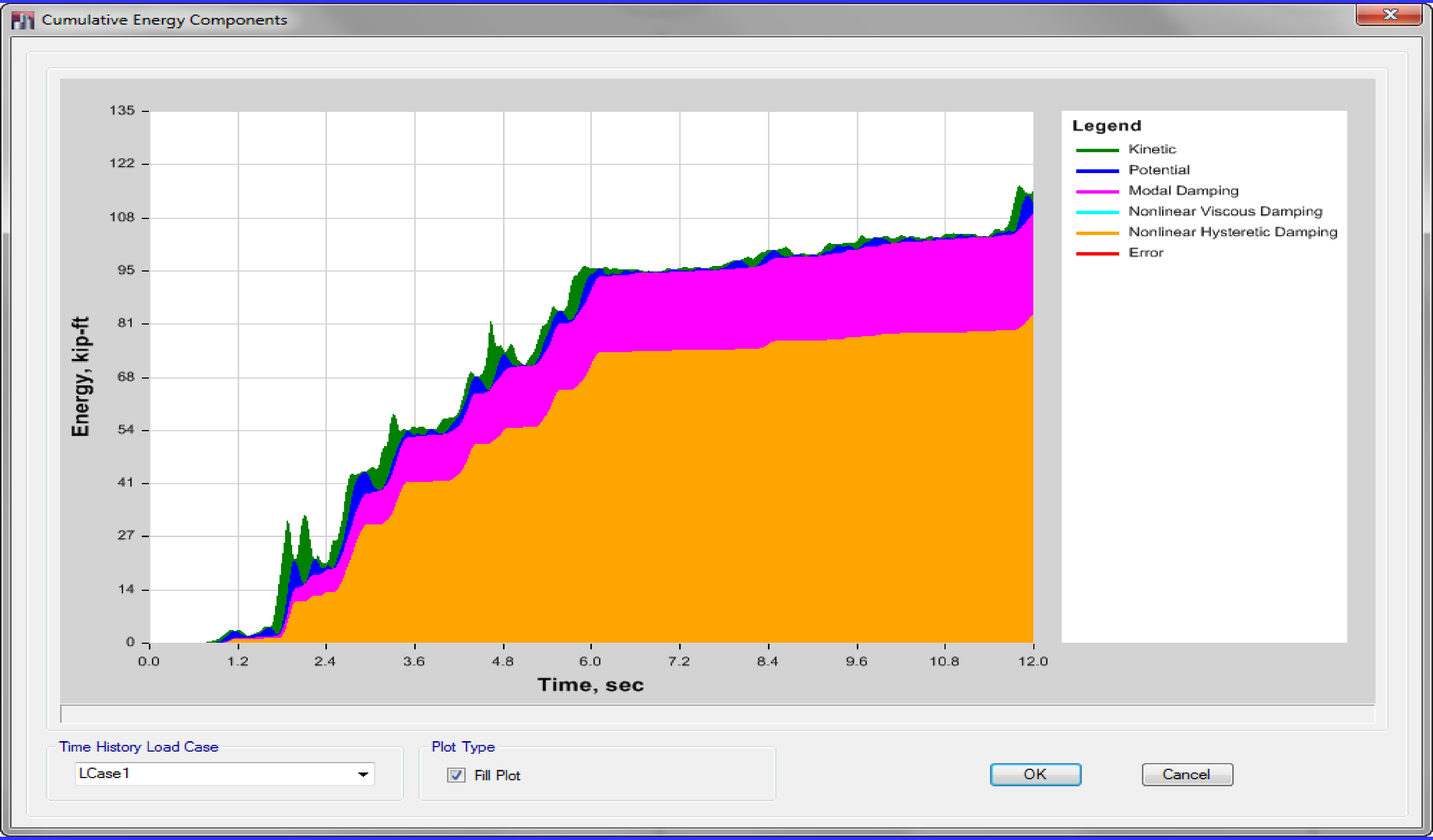
$$F^{(i)} = B^T f(t)^{(i)}$$

***Calculate error for iteration i , at the end of each time step, for the N Nonlinear elements – given Tolerance***

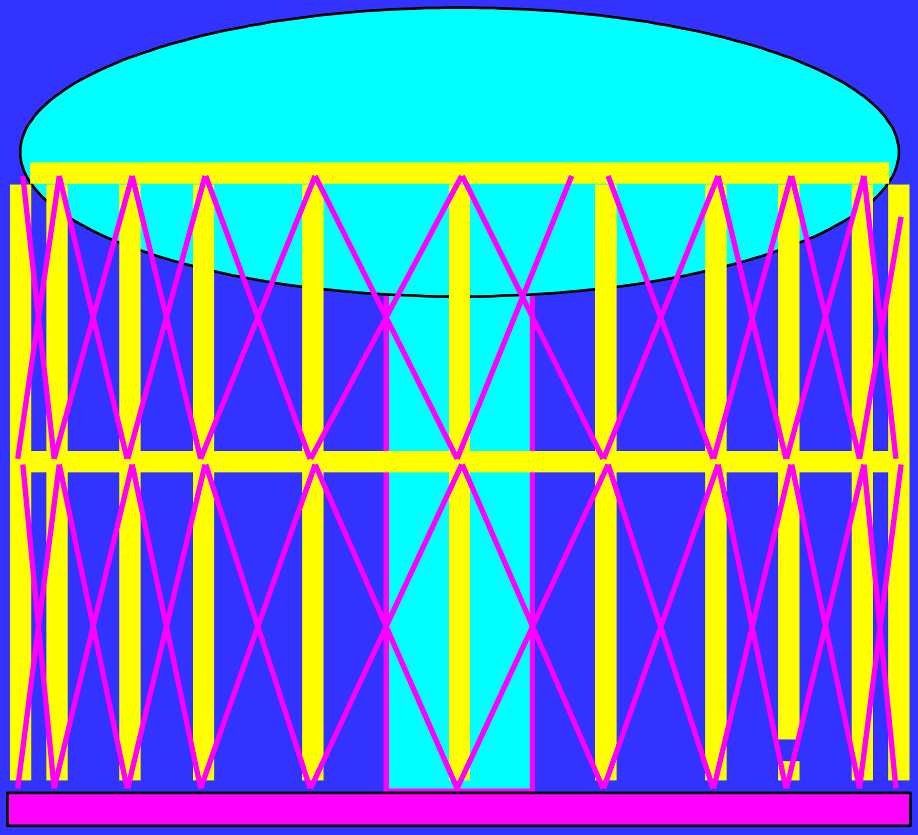
$$Err = \frac{\sum_{n=1}^{N} |\, \overline{f}(t)_n^{i} \,| - \sum_{n=1}^{N} |\, \overline{f}(t)_n^{i-1} \,|}{\sum_{n=1}^{N} |\, \overline{f}(t)_n^{i} \,|}$$

$$If \quad Err > Tol \; Continue to \; iterate \; with \; i - i + 1$$

$$If \quad Err < Tol \; go \; to \; next \; time \; step \; with \; t = t + \Delta t$$


Cumulative Energy Components
Energy, kip-ft
135
122
108
95
81
68
54
41
27
14
0
0.0
1.2
2.4
3.6
4.8
6.0
7.2
8.4
9.6
10.8
12.0
Time, sec
Legend
Kinetic
Potential
Modal Damping
Nonlinear Viscous Damping
Nonlinear Hysteretic Damping
Error
Time History Load Case
LCase1
Plot Type
Fill Plot
OK
Cancel

103 FEET DIAMETER - 100 FEET HEIGHT




# ELEVATED WATER STORAGE TANK

# COMPUTER MODEL

92 NODES – Over 500 DOF

103 ELASTIC FRAME ELEMENTS

56 NONLINEAR DIAGONAL ELEMENTS

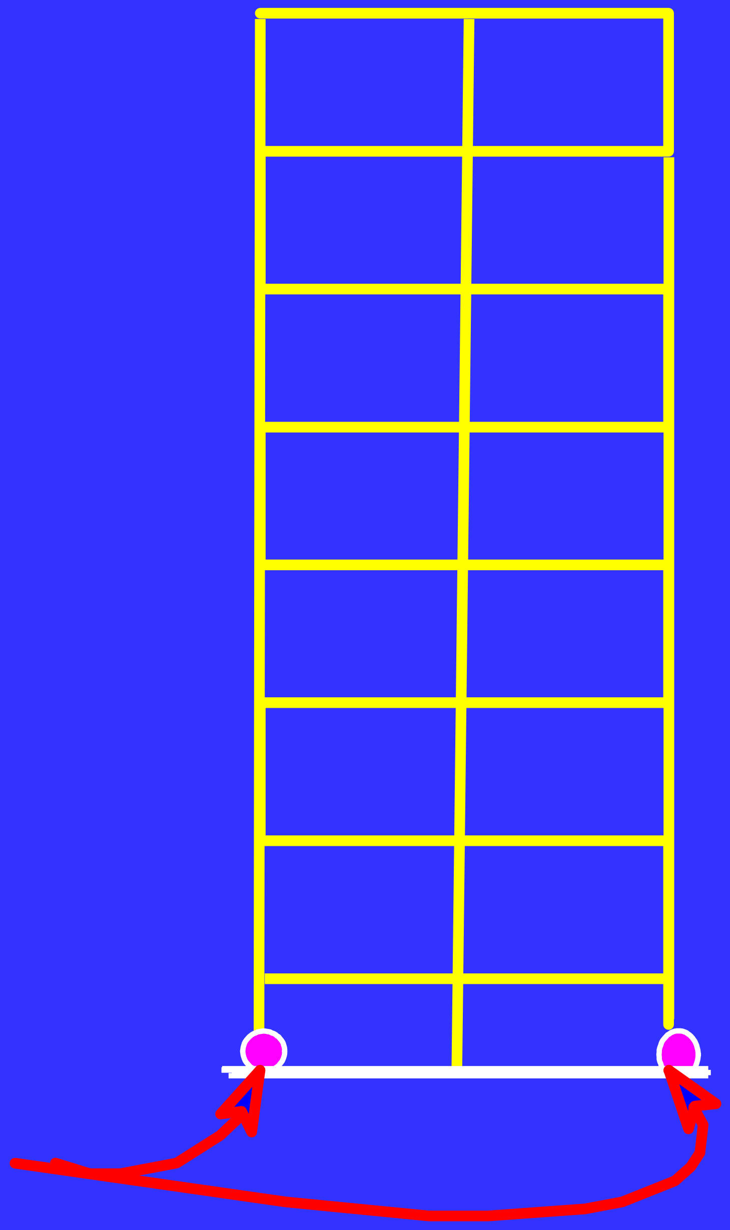
600 TIME STEPS @ 0.02 Seconds

# COMPUTER TIME REQUIREMENTS

| PROGRAM | | | |
|---|---|---|---|
| ANSYS | INTEL 486 | 3 Days | ( 4300 Minutes ) |
| ANSYS | CRAY | 3 Hours | ( 180 Minutes ) |
| SADSAP | INTEL 486 | | (2 Minutes ) |

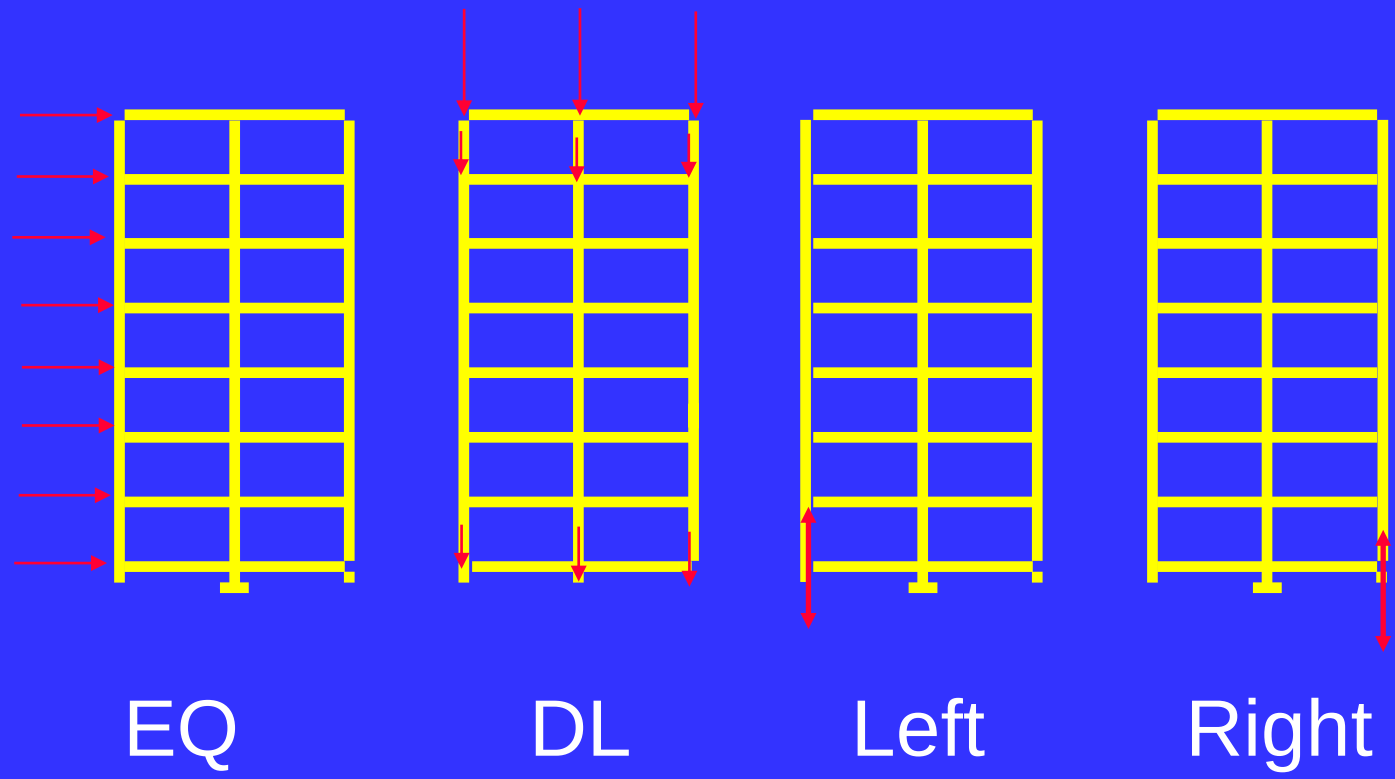
**( B Array was 56 x 20 )**

# FRAME WITH UPLIFTING ALLOWED



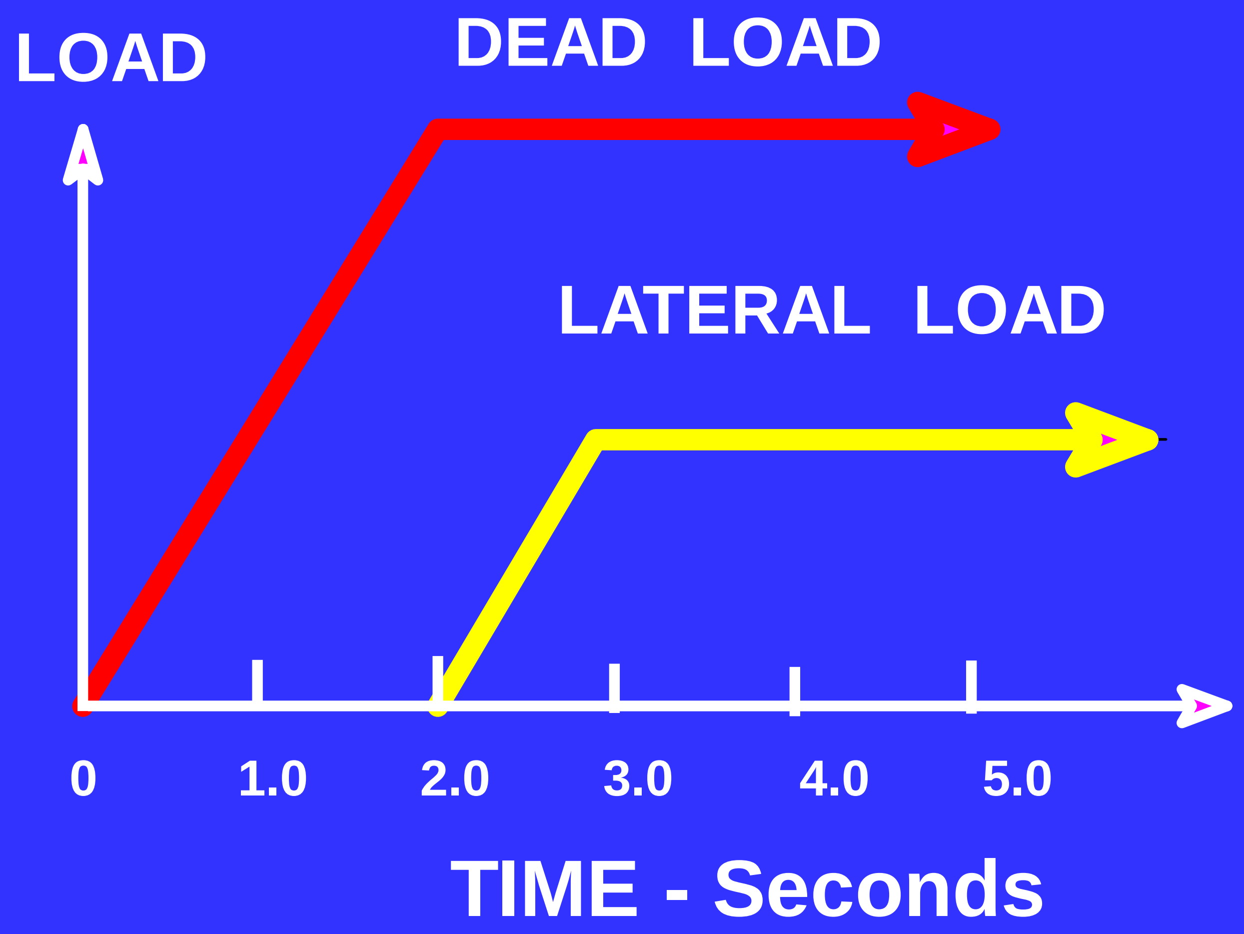
UPLIFTING ALLOWED

# *Four Static Load Conditions Are Used To Start The Generation of LDR Vectors*

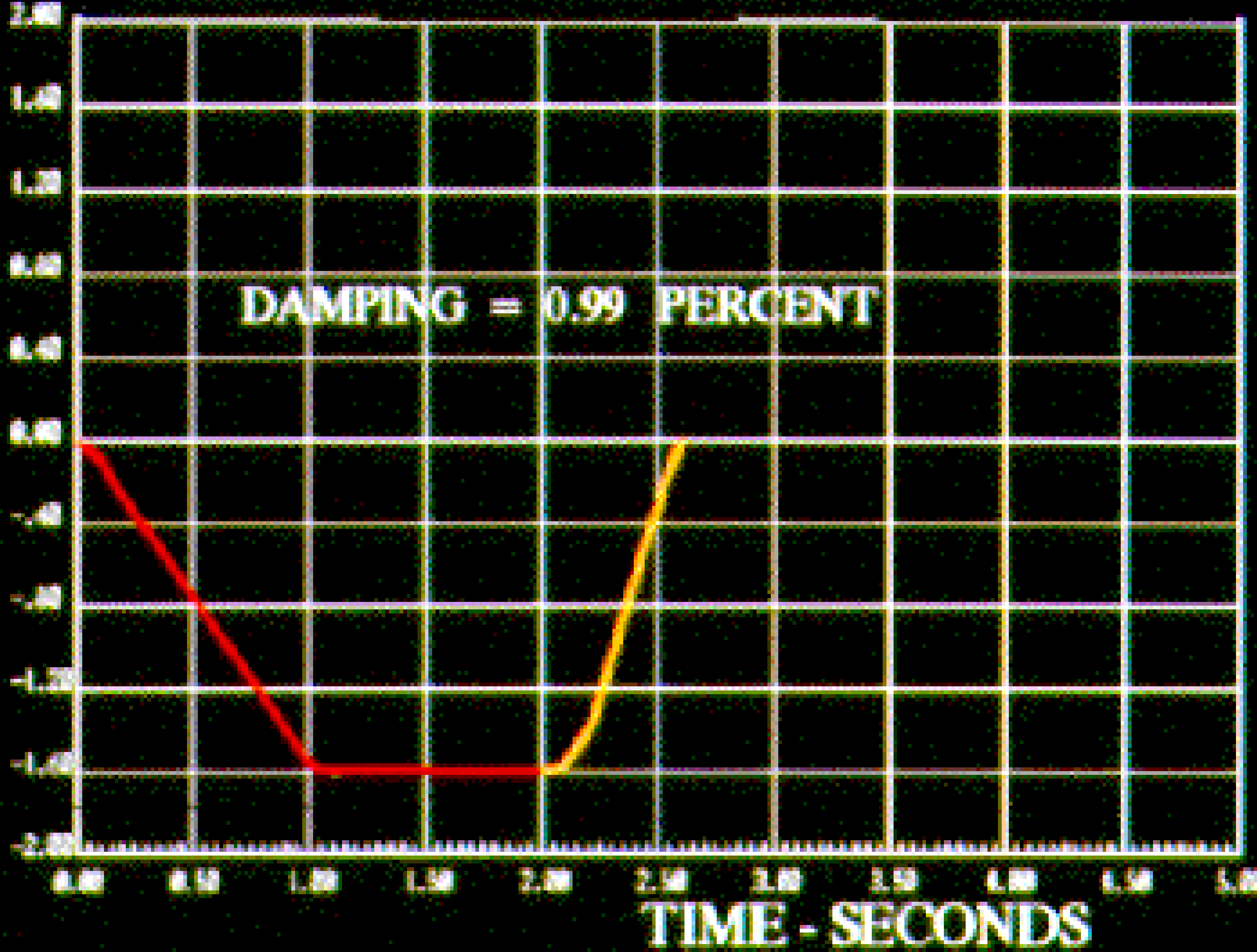


# NONLINEAR STATIC ANALYSIS

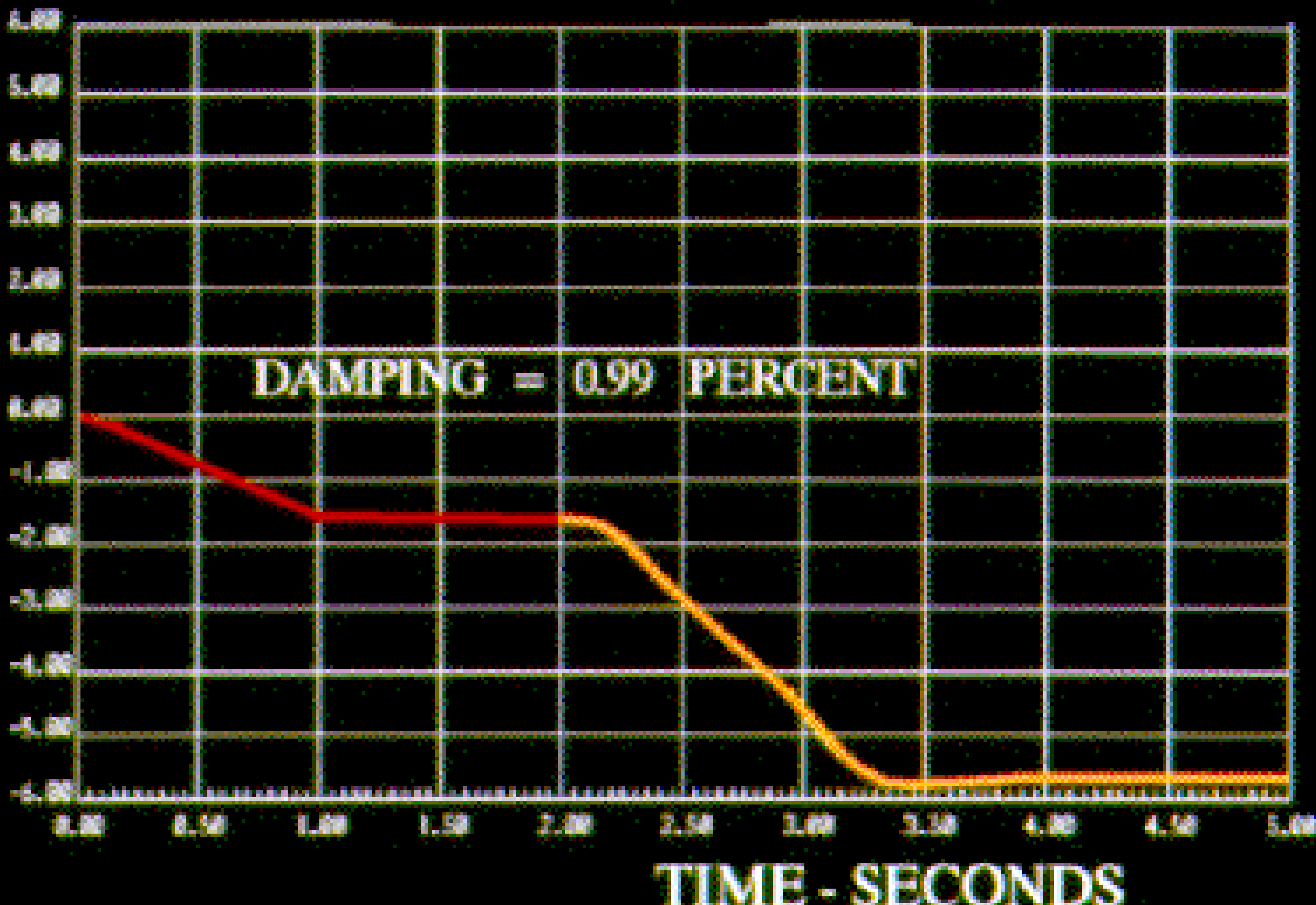
50 STEPS AT dT = 0.10 SECONDS

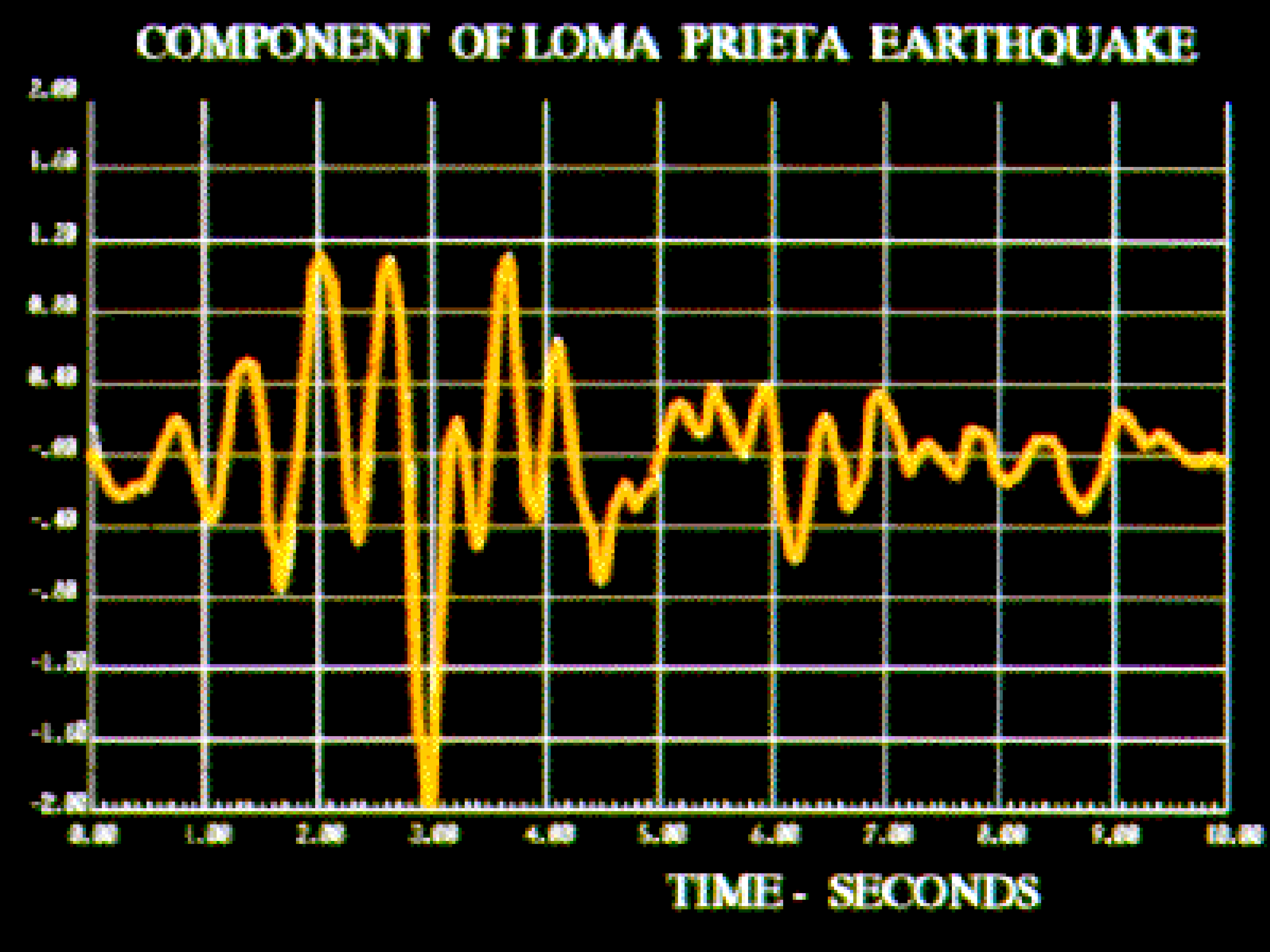


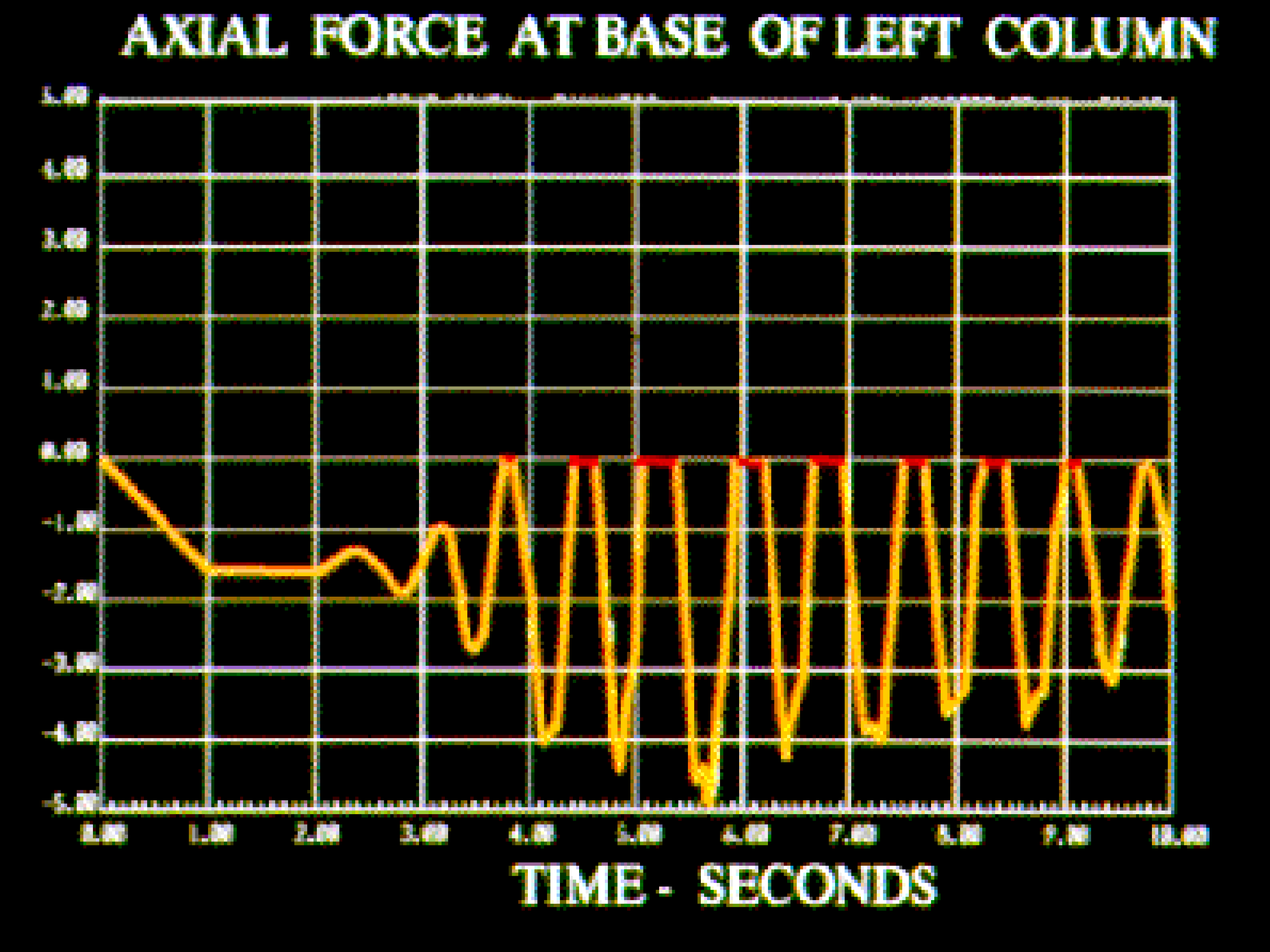
# FORCE AT BASE OF LEFT COLUMN



# FORCE AT BASE OF RIGHT COLUMN




COMPONENT OF LOMA PRIETA EARTHQUAKE
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
TIME - SECONDS


AXIAL FORCE AT BASE OF LEFT COLUMN
5.00
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
-4.00
-5.00
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
TIME - SECONDS

# Axial Forces in Left and Right Columns as a Function of Time

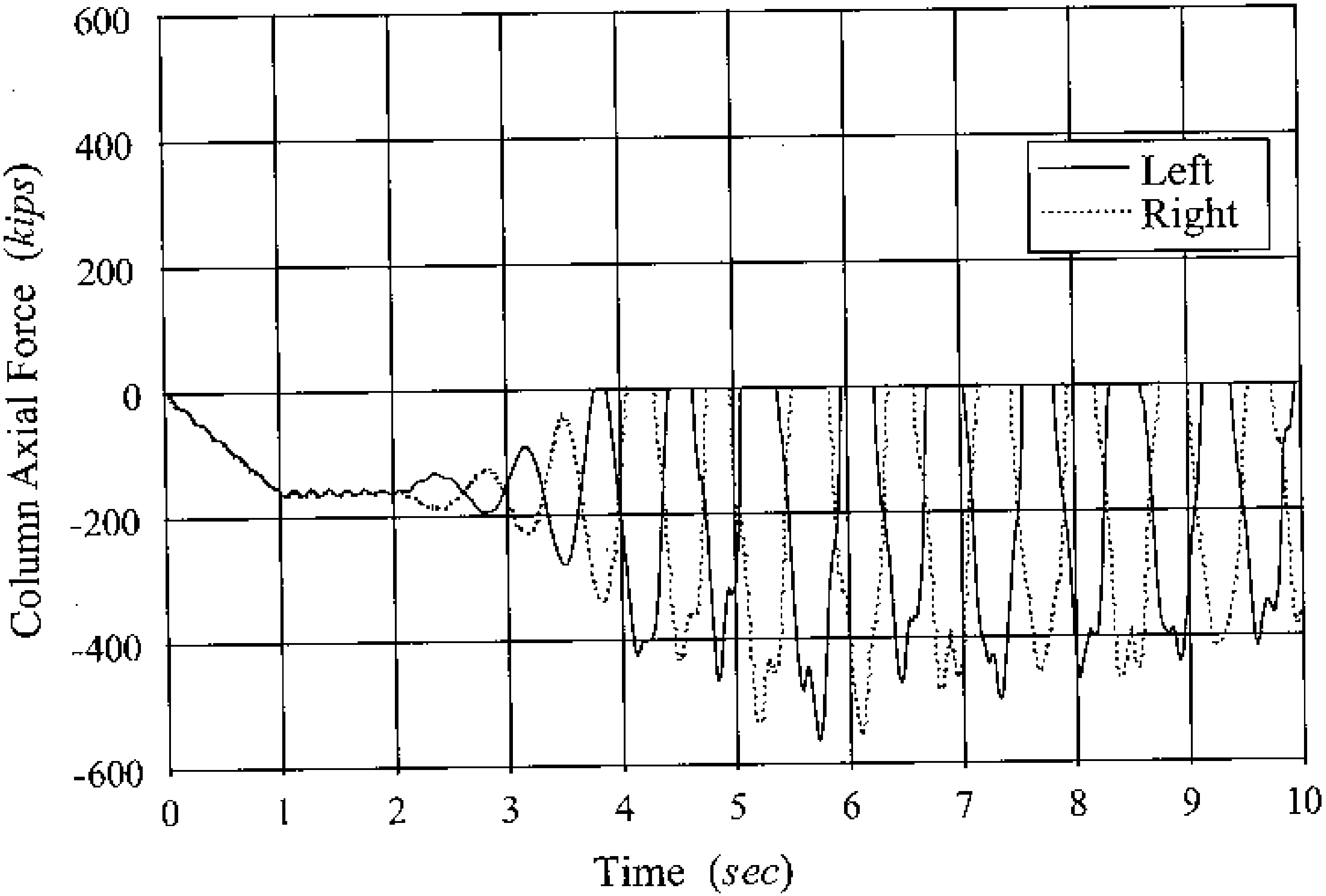



**Figure 18.7:** Column Axial Forces from Earthquake Loading

# Results With and Without Uplift Loma Prieta Earthquake

| Response Type | Response With or Without Uplift | | Difference (*percent*) |
|---|---|---|---|
| | With | Without | |
| Maximum Displacement (*in*) | 3.90 | 3.88 | +0.5 |
| Maximum Axial Force (*kips*) | 505 | 542 | −6.8 |
| Maximum Base Shear (*kips*) | 199 | 247 | −19.4 |
| Maximum Base Moment (*kip-in*) | 153,000 | 212,000 | −27.8 |
| Maximum Strain Energy (*kip-in*) | 428 | 447 | −4.2 |
| Computational Time (*sec*) | 15 | 14.6 | +3.0 |

**Table 18.2:** Summary of Results for Building Uplifting Problem from the Loma Prieta Earthquake ($\xi = 0.05$)

# *Results With and Without Uplift 2 Times the Loma Prieta Earthquake*

| Response Type | Response With or Without Uplift | | Difference (*percent*) |
|---|---|---|---|
| | With | Without | |
| Maximum Displacement (*in*) | 5.88 | 7.76 | −24 |
| Maximum Axial Force (*kips*) | 620 | 924 | −33 |
| Maximum Base Shear (*kips*) | 255 | 494 | −40 |
| Maximum Base Moment (*kip-in*) | 197,000 | 424,000 | −53 |
| Maximum Strain Energy (*kip-in*) | 489 | 1,547 | −68 |
| Computational Time (*sec*) | 1.16 | ....... | .... |

**Table 18.3:** Summary of Results for Building Uplifting Problem from Two Times the Loma Prieta Earthquake ($\xi = 0.05$)

# *Advantages Of The FNA Method*

1. *The Method Can Be Used For Both Static And Dynamic Nonlinear Analyses*

2. *The Method Is Very Efficient And Requires A Small Amount Of Additional Computer Time As Compared To Linear Analysis*

2. *The Method Can Easily Be Incorporated Into Existing Computer Programs For LINEAR DYNAMIC ANALYSIS.*

# *FIRST LARGE APPLICTION OF THE FNA METHOD*

***Retrofit of the***

***RICHMOND - SAN RAFAEL BRIDGE***

***1997 to 2000***

***Using SADSAP***

**S** TATIC

**A** ND

**D** YNAMIC

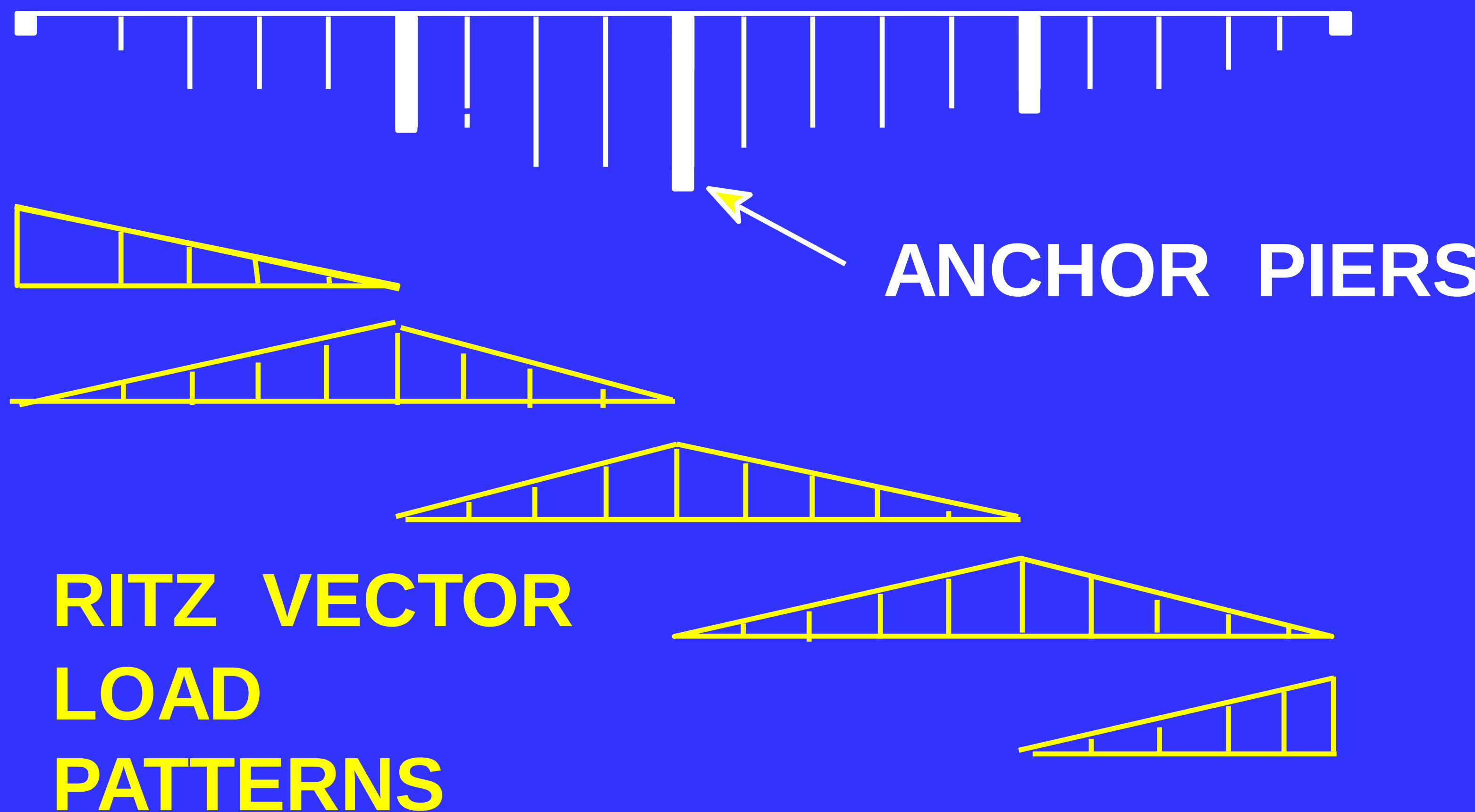
**S** TRUCTURAL

**A** NALYSIS

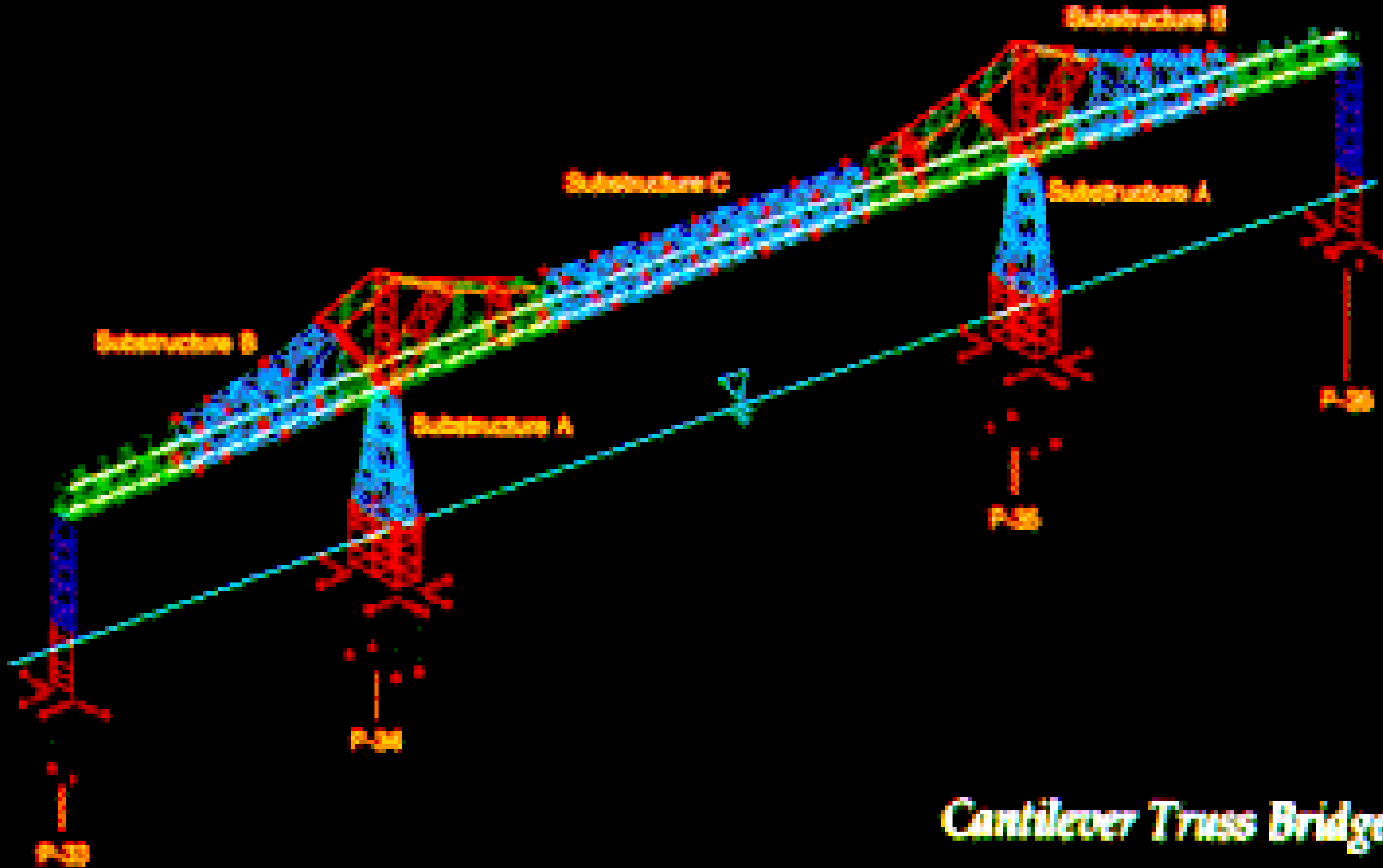
**P** ROGRAM

# MULTISUPPORT ANALYSIS

## ( Displacements )



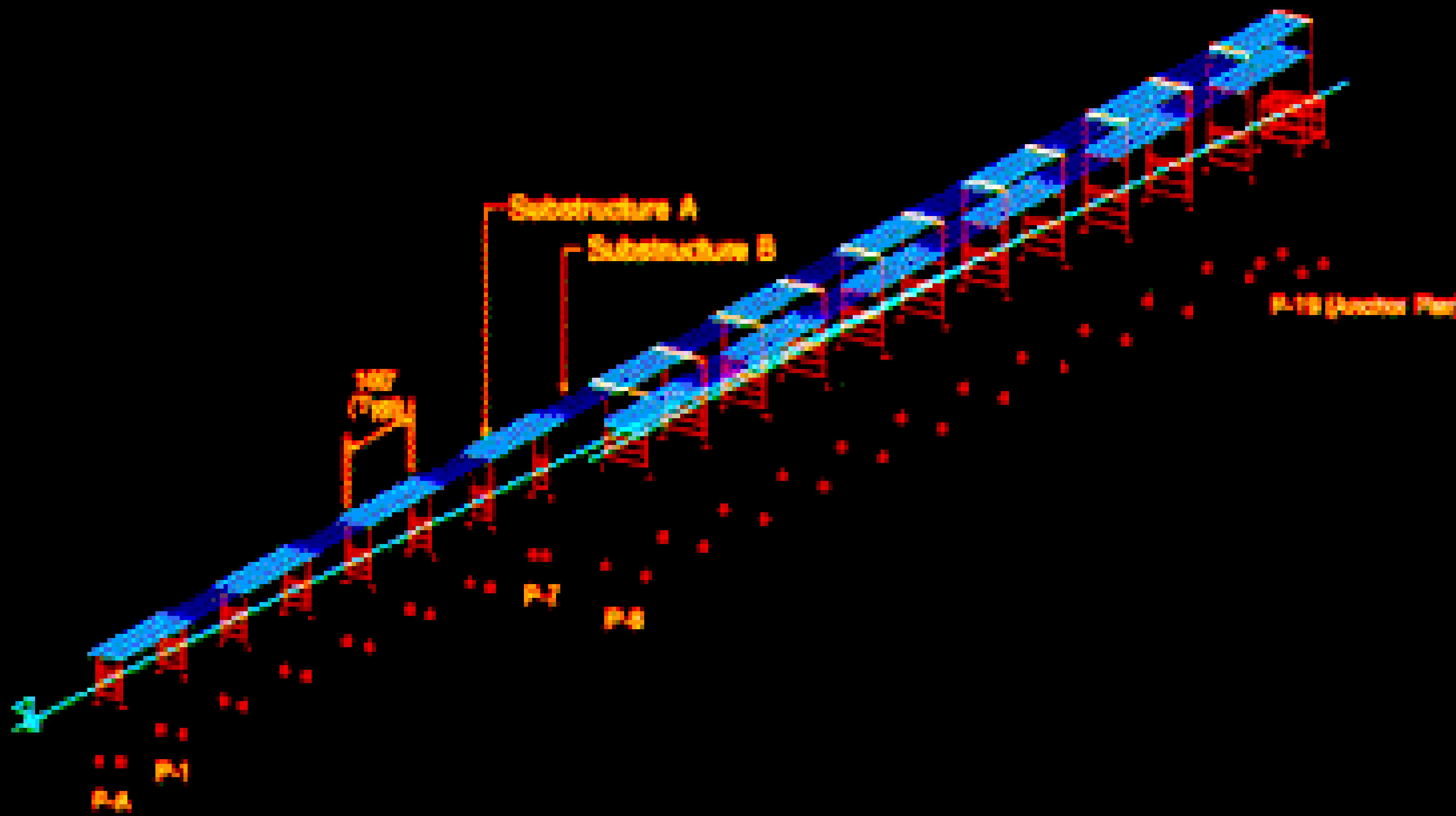
# Sub-Structured Model




*Cantilever Truss Bridge*

ICF KAISER

# Richmond-San Rafael Bridge

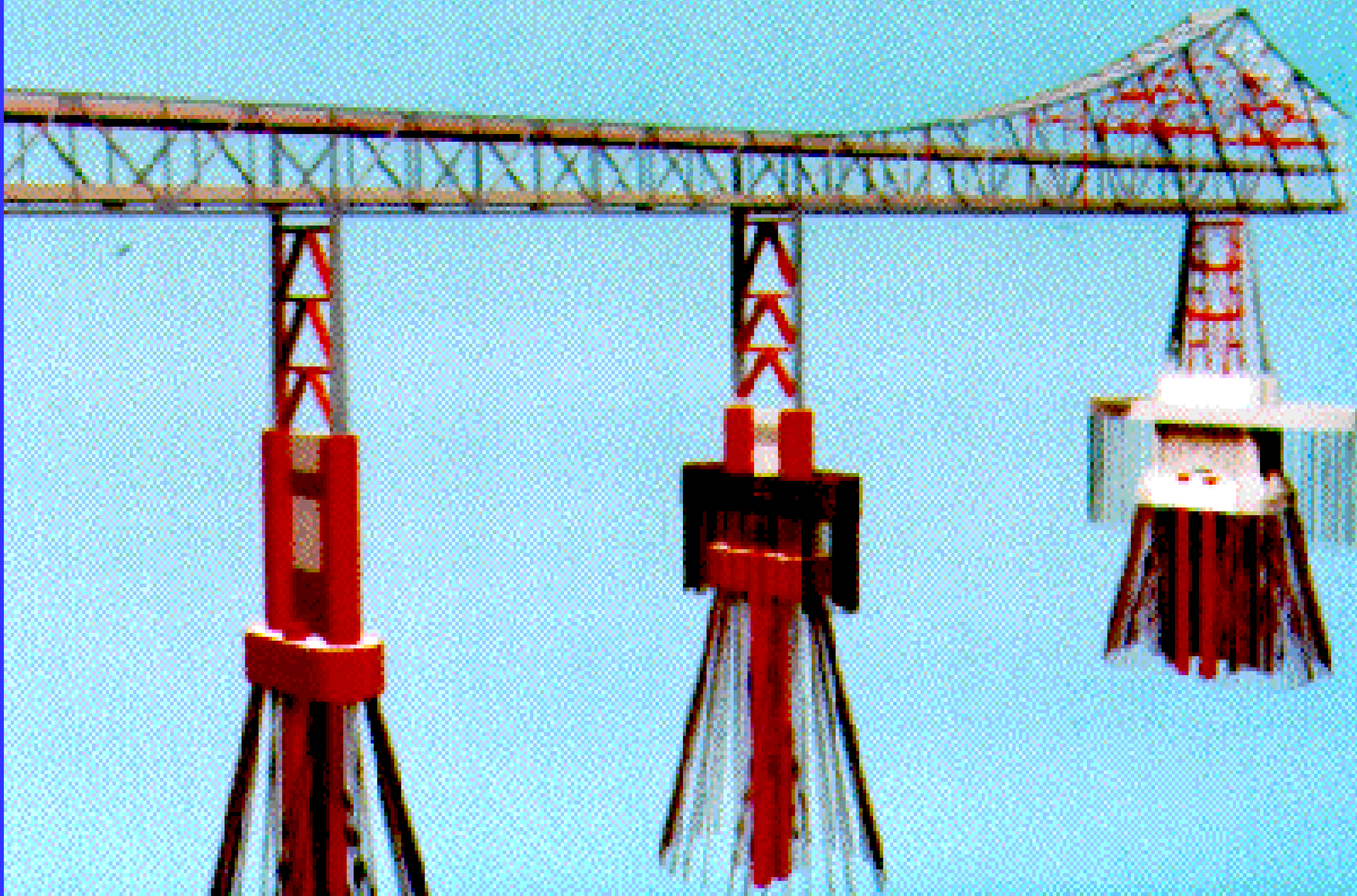



*Seg-A/Plate Girder Spans*

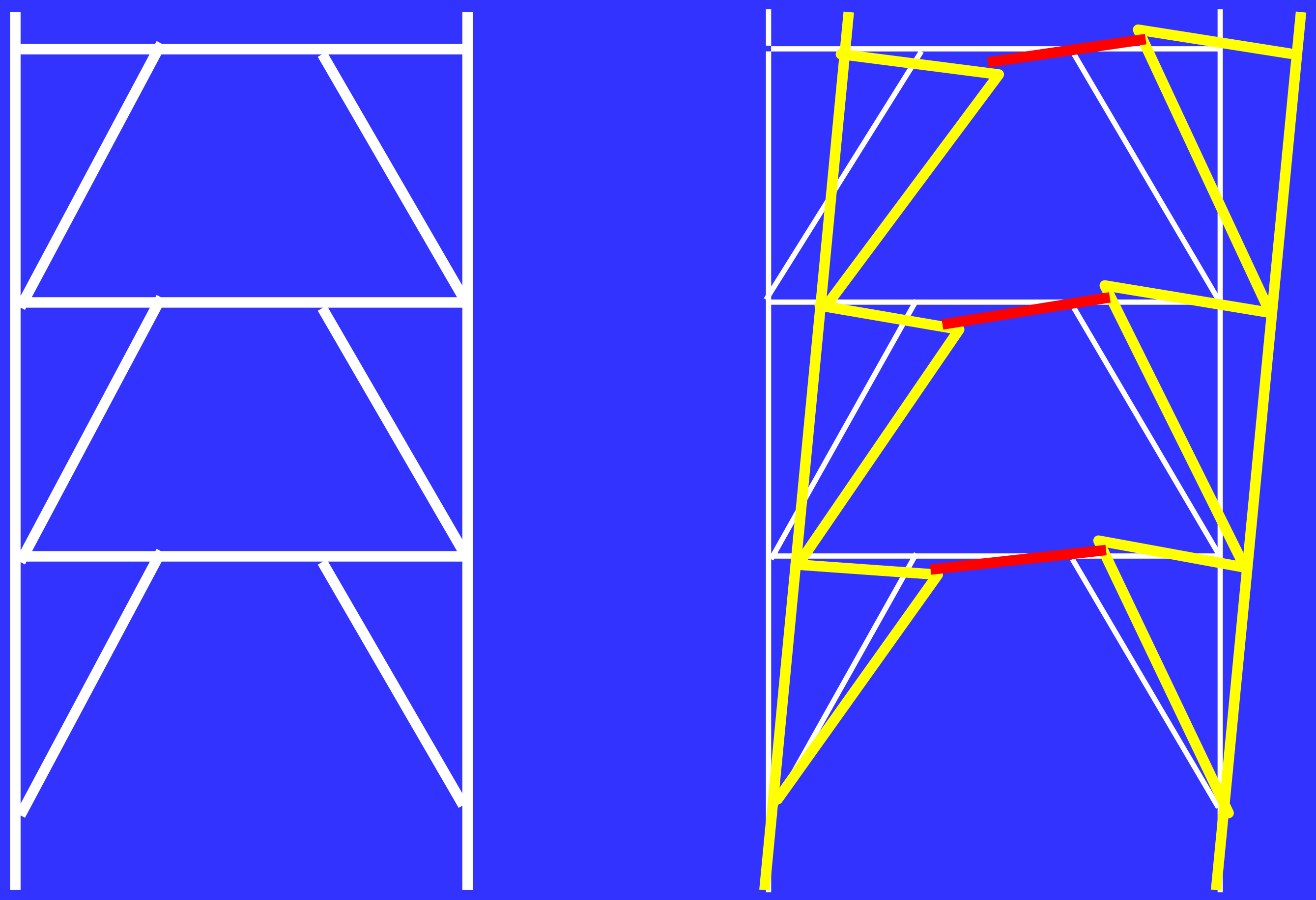

RICHMOND - SAN RAFAEL BRIDGE
AS BUILT

# RICHMOND - SAN RAFAEL BRIDGE

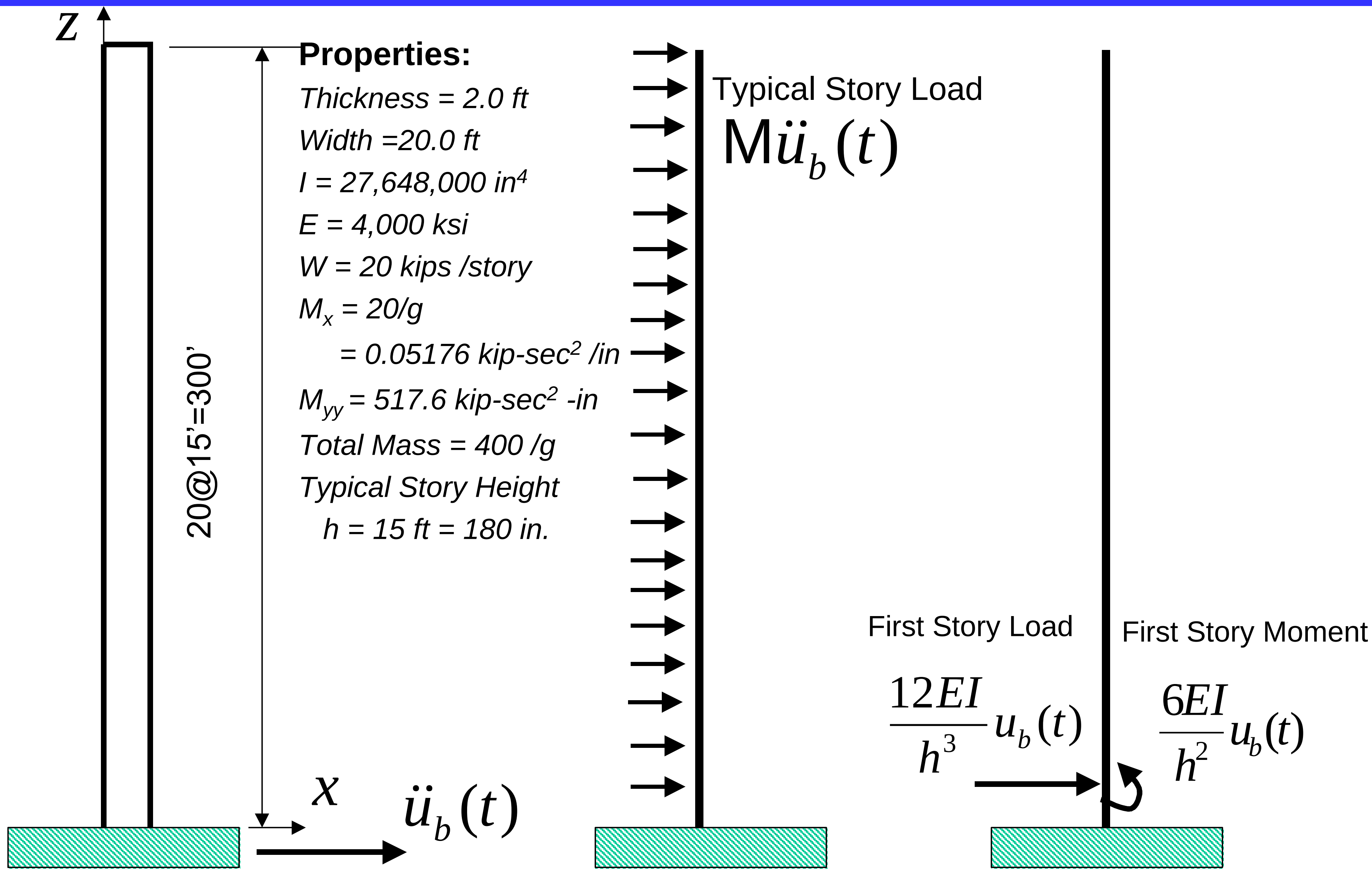
## RETROFIT ELEMENTS



# ECCENTRICALLY BRACED FRAME



# Comparison of Relative and Absolute Displacement Seismic Analysis

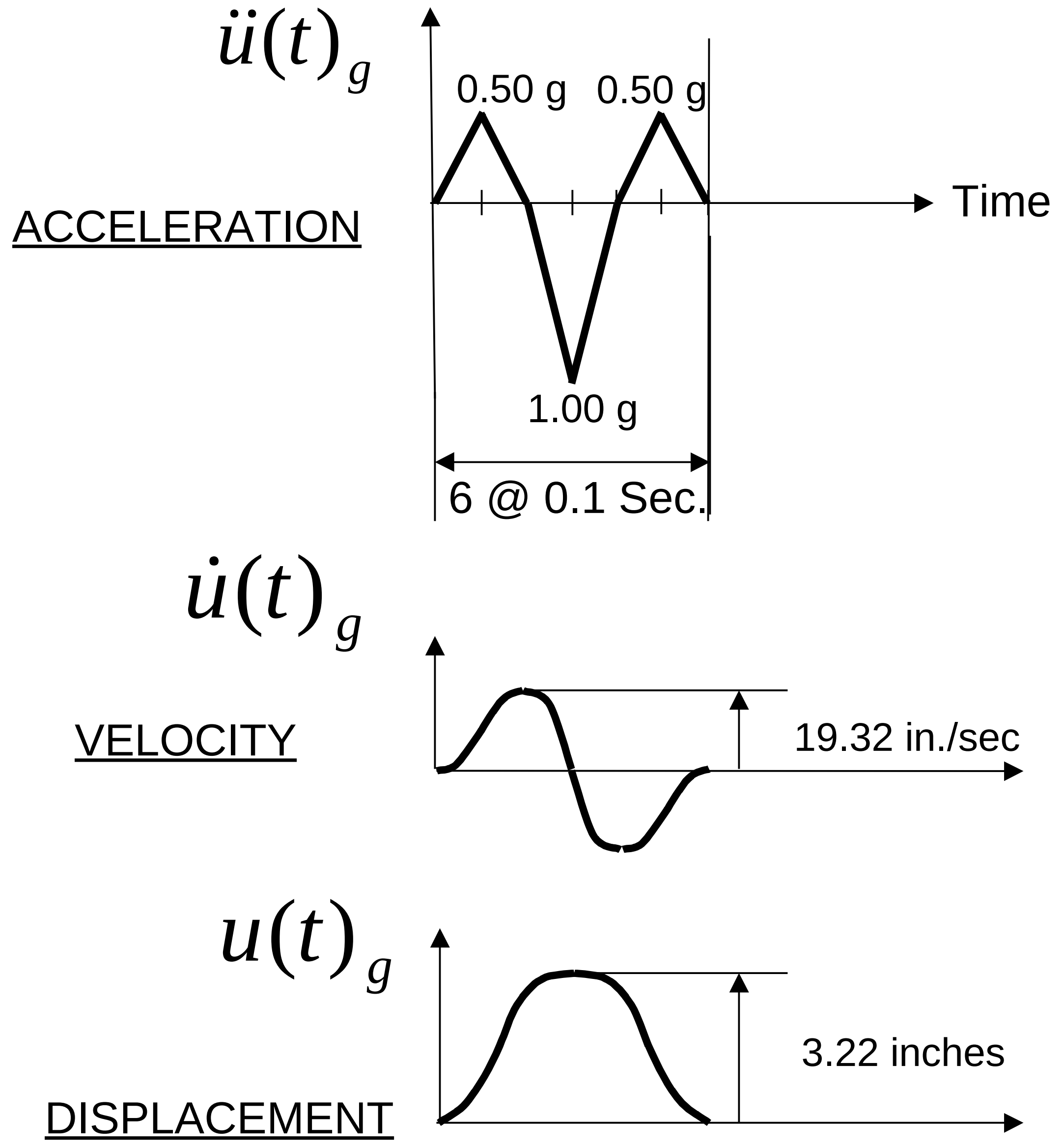



A. 20 Story Shear Wall
With Story Mass

B. Base Acceleration Loads
Relative Formulation

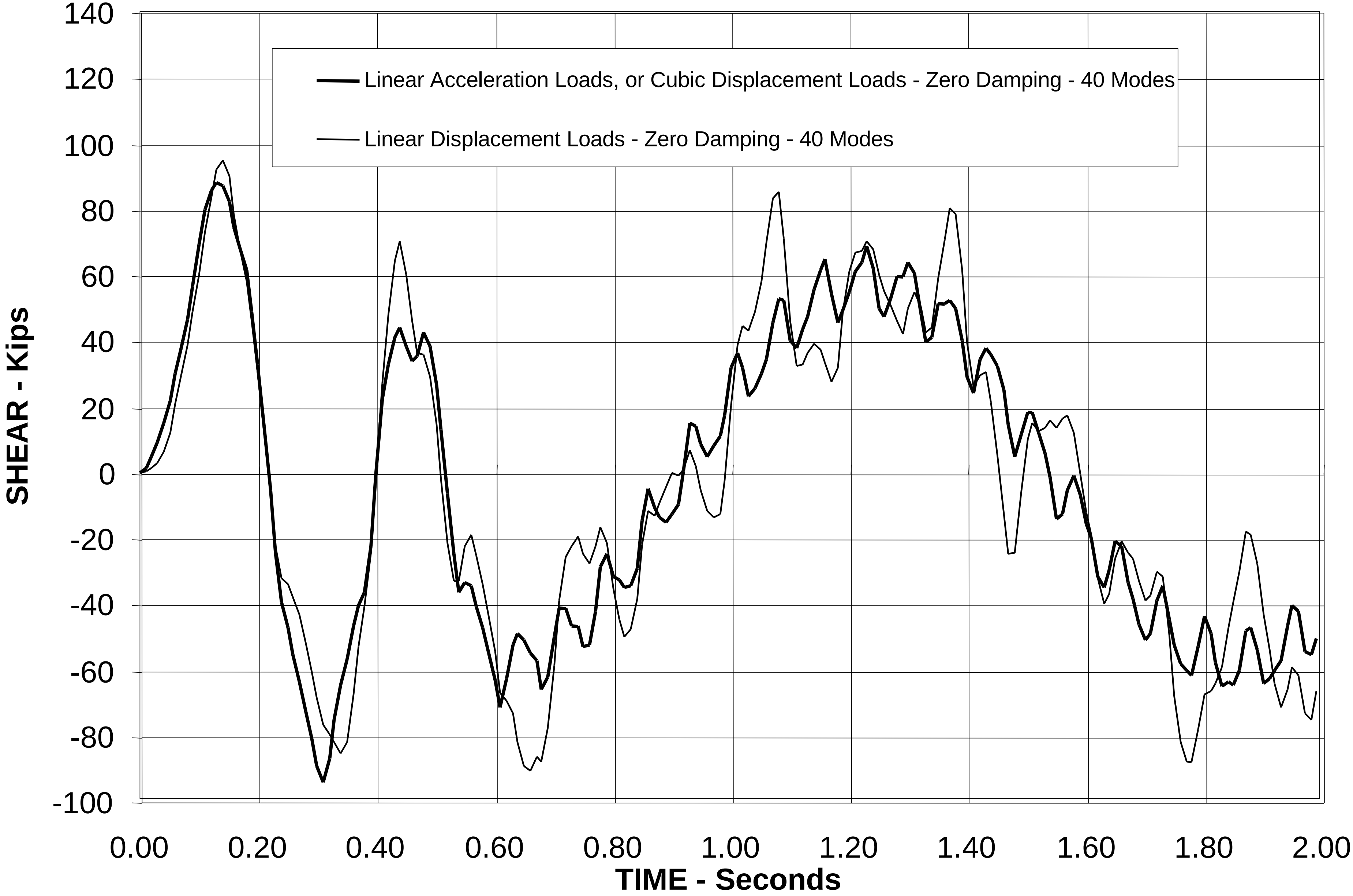
C. Displacement Loads
Absolute Formulation

# Idealized Near-Field Earthquake Motions

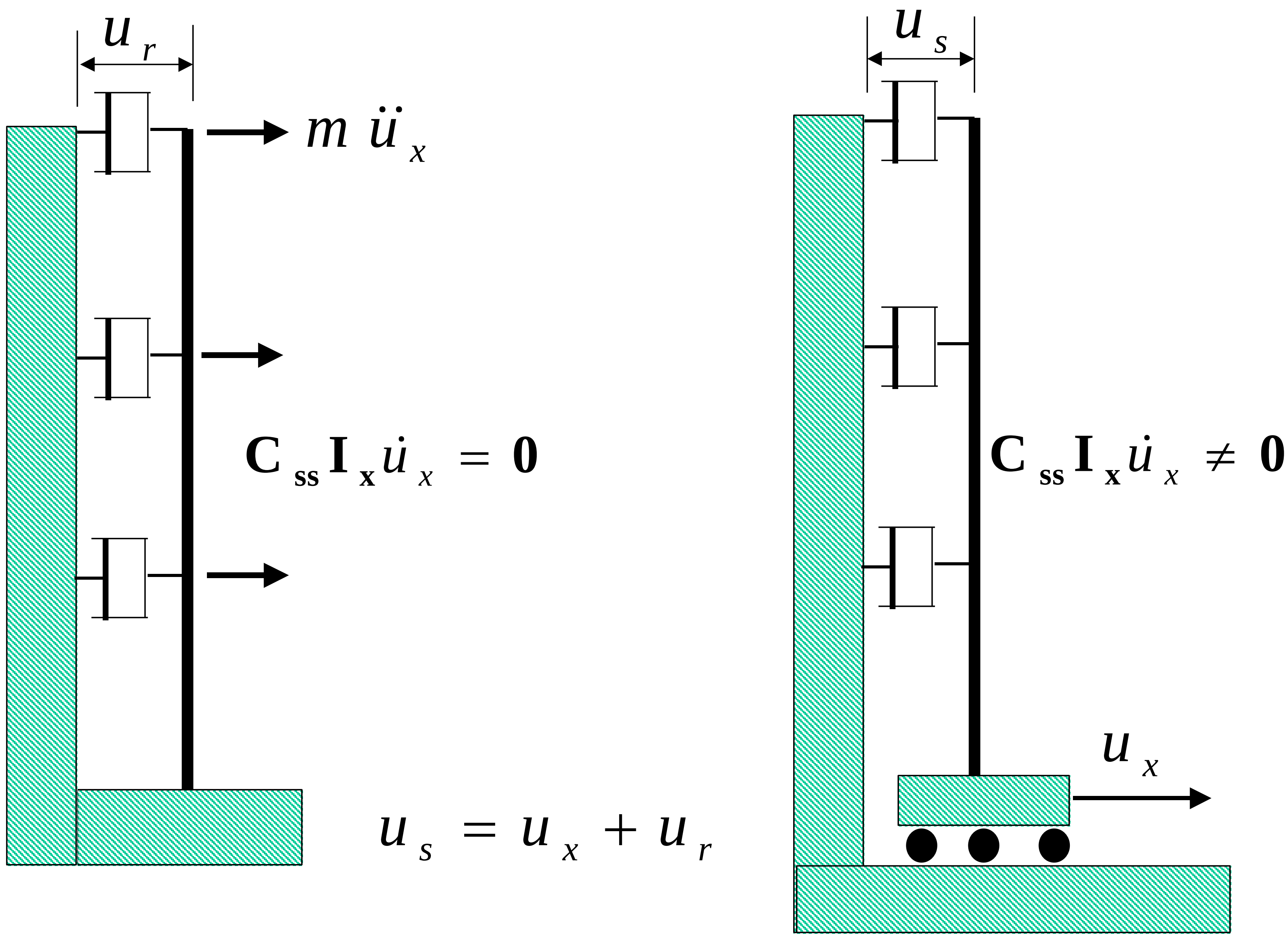


# Shear at Second Level Vs. Time With Zero Damping

## Time Step = 0.01



# *Illustration of Mass-Proportional Component in Classical Damping.*




RELATIVE DISPLACEMENT FORMULATION

ABSOLUTE DISPLACEMENT FORMULATION

# *Comparison of Mode Superposition of LDR Vectors with the Step-By-Step Solution using the Trapezoidal Rule*

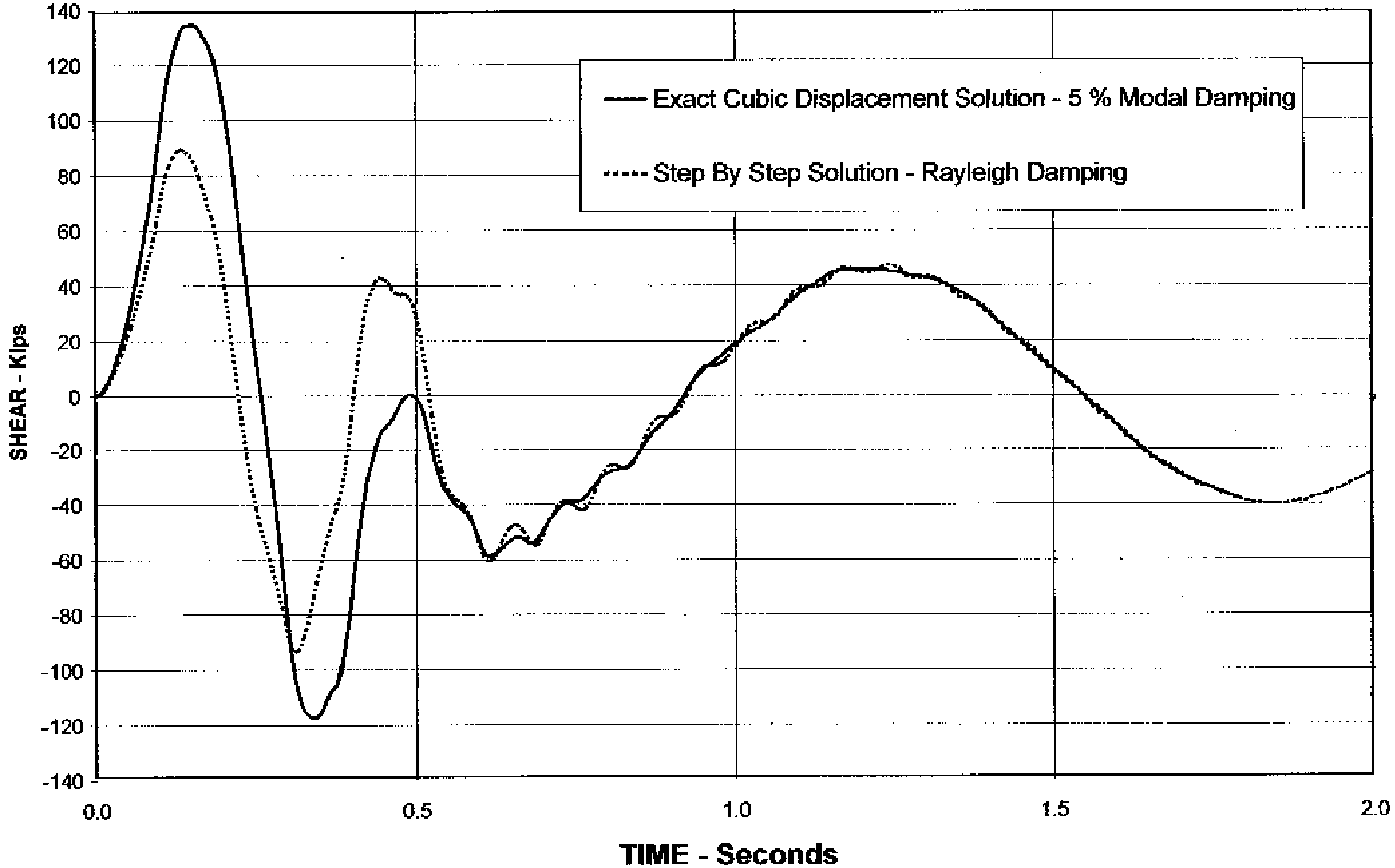



***Figure 22.7 Comparison of Step-By-Step Solution Using the Trapezoidal Rule and Rayleigh Damping with Exact Solution (0.005 second time-step and 5% damping)***

# A Fluid is a Solid with a Very Low Shear Modulus

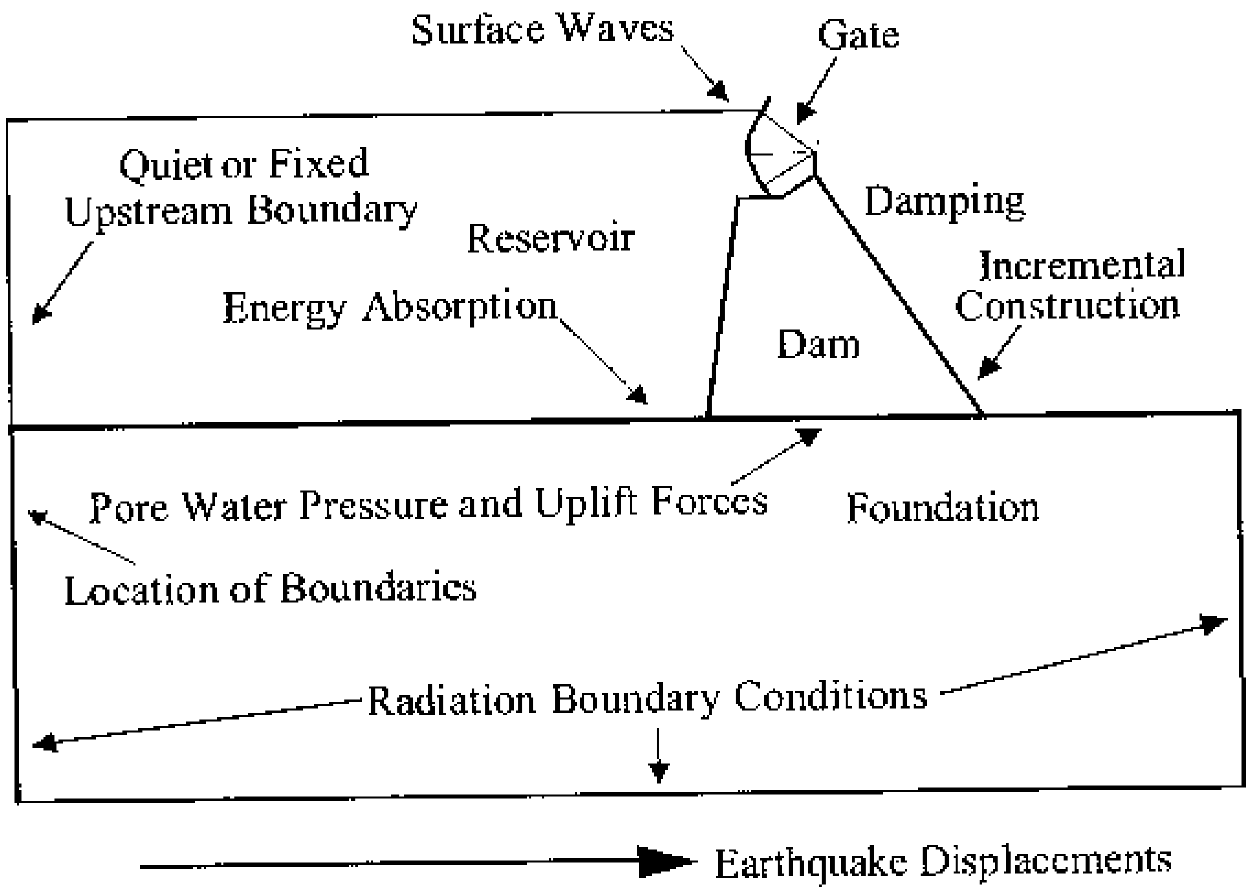



**Figure 23.1:** Approximations Required for the Creation of a Finite Element Model For Fluid/Solid Systems

# Recommendations

*Use realistic physical approximations to create the mathematical model . Do parameter studies to investigate the sensitivity when it is possible. Find out "if it matters".*

*Structural and Geotechnical Engineers must work together to produce "a family" of realistic Earthquake records. Do not accept a site Spectra only.*

*After the model is created and the loading selected, use the most accurate solution methods available. Computer time is free.*

# Ed’s Simple Consideration

*After you understand the behavior of the structure, take time to consider several different options to improve the earthquake resistance  of the structure.*

*It is my feeling there are two fundamental approaches:*

. *Make the structure stiffer to move as a rigid body.*

. *Make the structure more flexible at the base to isolate it without causing irreparable damage.*

*The solution must prevent collapse and  be cost effective. Also, the damage must be repairable.*

*You and SAP 2000*

*Can do your own Research*