# Demand/Capacity Ratios Calculations for 3D Frame Elements

## THE EQUATION

Many building codes specify that all one-dimensional **_compression_** members within a structure satisfy the following Demand/Capacity Ratio at all points in time:

$$R(t) = \frac{|P(t)|}{f_c P_{cr}} \pm \frac{M_2(t)C_2}{f_b M_{c2}(1 - \frac{|P(t)|}{P_{e2}})} \pm \frac{M_3(t)C_3}{f_b M_{c3}(1 - \frac{|P(t)|}{P_{e3}})} \leq 1.0$$

Where the forces acting on the frame element cross-section at time "t" are $P(t), M_2(t)$ and $M_3(t)$ (including the static forces prior to the application of the dynamic loads). The empirical constants are code and material dependent and are normally defined as

| | | |
|---|---|---|
| $f_c$ and $f_b$ | = | Resistance factors |
| $C_2$ and $C_3$ | = | Moment reduction factors |
| $M_{c2}$ and $M_{c3}$ = | | Moment capacity |
| $P_{cr}$ | = | Axial load Capacity |
| $P_{e2}$ and $P_{e3}$ | = | Euler bucking load capacity about the 2 and 3 axis with effective length approximated. |

These equations have been used for many years for static load analyses. For each **_time-history seismic analysis_**, $P(t), M_2(t)$ and $M_3(t)$ at every cross-section of all members can be easily calculated as a function of time. Therefore, the maximum Demand/Capacity Ratio, $R(t)$ for all load conditions, can be accurately calculated and identified by any modern computer analysis design program in a fraction of a second. All of the CSI series of programs have this capability built into their interactive post-processing programs.

For each **_response spectrum analysis,_** however, the value of $P(t), M_2(t)$ and $M_3(t)$ cannot be calculated accurately since only positive values of $P, M_2$ and $M_3$ are produced. These are peak maximum values have a very low probability of occurring at the same time. Therefore, the Demand/Capacity Ratios are always significantly greater than those produced by a time-history analysis.

The author, acting as a consultant on the retrofit of the San Mateo Bridge after the Loma Prieta Earthquake, has had significant experience with the problem of calculating Demand/Capacity Ratios using the response spectra method. The seismologists and geotechnical engineers created several different sets of three-dimensional ground motions. They generated both near and far field motions from both the Hayward and San Adreas faults. Then, they averaged the various ground motions and produced three-dimensional design spectra to be used to design the retrofit of the Bridge.

The structural engineering group that tried to use the design spectra for the analysis and retrofit of the bridge found that a large number of members in the structure required retrofit. After a careful study of the maximum peak values of the member forces (especially the large peak axial forces), it was decided to run new time-history analyses using the basic time-history records that were used to create the design spectra. After running all the time-history records, the maximum **_Demand/Capacity Ratios were reduced by approximately a factor of three compared to the design spectra results_**.

### *OTHER COMMON ERRORS IN THE EVAUATION OF THE DEMAND/CAPACITY EQUATION*

Veletsos and Newmark, at the 2rd WCEE in Tokyo in January 1960, concluded the dynamic displacements obtained from a nonlinear time-history earthquake analysis of a single degree of freedom system were approximately the same as the displacements obtained from a linear elastic dynamic analysis of the same system.

This *Equal Displacement Rule* was proven to be incorrect when Ray Clough presented the paper **"Inelastic Earthquake Response of Tall Buildings"** at the 3rd World Conference on Earthquake Engineering in January 1965 which was held in New Zealand. This lost paper can be read and printed from the following link:

[http://edwilson.org/History/History%20of%20Nonlinear%20Seismic%20Analysis.pdf](http://edwilson.org/History/History%20of%20Nonlinear%20Seismic%20Analysis.pdf)

CalTrans and many other structural engineering organizations have continued to use this old and incorrect rule for the past 54 years. In addition, at the present time it is used to predict the safety of many tall structures by using one very simple pushover analysis.