2.

EQUILIBRIUM AND COMPATIBILITY

Equilibrium Is Essential - Compatibility Is Optional

2.1 INTRODUCTION

Equilibrium equations set the externally applied loads equal to the sum of the internal element forces at all joints or node points of a structural system; they are the most fundamental equations in structural analysis and design. The exact solution for a problem in solid mechanics requires that the differential equations of equilibrium for all infinitesimal elements within the solid must be satisfied. *Equilibrium is a fundamental law of physics and cannot be violated within a "real" structural system.* Therefore, it is critical that the mathematical model, which is used to simulate the behavior of a real structure, also satisfies those basic equilibrium equations.

It is important to note that within a finite element, which is based on a formal displacement formulation, the differential stress-equilibrium equations are not always satisfied. However, inter-element force-equilibrium equations are identically satisfied at all node points (joints). The computer program user who does not understand the approximations used to develop a finite element can obtain results that are in significant error if the element mesh is not sufficiently fine in areas of stress concentration[1].

Compatibility requirements should be satisfied. However, if one has a choice between satisfying equilibrium or compatibility, one should use the equilibrium-based solution. For real nonlinear structures, equilibrium is always satisfied in

the deformed position. Many real structures do not satisfy compatibility caused by creep, joint slippage, incremental construction and directional yielding.

2.2 FUNDAMENTAL EQUILIBRIUM EQUATIONS

The three-dimensional equilibrium of an infinitesimal element, shown in Figure 1.1, is given by the following equilibrium equations[2]:

$$\frac{\partial \sigma_1}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} + \frac{\partial \tau_{13}}{\partial x_3} + \beta_1 = 0$$

$$\frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \sigma_2}{\partial x_2} + \frac{\partial \tau_{23}}{\partial x_3} + \beta_2 = 0 \qquad (2.1)$$

$$\frac{\partial \tau_{31}}{\partial x_1} + \frac{\partial \tau_{32}}{\partial x_2} + \frac{\partial \sigma_3}{\partial x_3} + \beta_3 = 0$$

The body force, β_i, is per unit of volume in the i-direction and represents gravitational forces or pore pressure gradients. Because $\tau_{ij} = \tau_{ji}$, the infinitesimal element is automatically in rotational equilibrium. Of course for this equation to be valid for large displacements, it must be satisfied in the deformed position, and all stresses must be defined as force per unit of deformed area.

2.3 STRESS RESULTANTS - FORCES AND MOMENTS

In structural analysis it is standard practice to write equilibrium equations in terms of stress resultants rather than in terms of stresses. Force stress resultants are calculated by the integration of normal or shear stresses acting on a surface. Moment stress resultants are the integration of stresses on a surface times a distance from an axis.

A point load, which is a stress resultant, is by definition an infinite stress times an infinitesimal area and is physically impossible on all real structures. Also, a point moment is a mathematical definition and does not have a unique stress field as a physical interpretation. Clearly, the use of forces and moments is fundamental in structural analysis and design. However, a clear understanding of their use in

finite element analysis is absolutely necessary if stress results are to be physically evaluated.

For a finite size element or joint, a substructure, or a complete structural system *the following six equilibrium equations must be satisfied:*

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{2.2}$$

For two dimensional structures only three of these equations need to be satisfied.

2.4 COMPATIBILITY REQUIREMENTS

For continuous solids we have defined strains as displacements per unit length. To calculate absolute displacements at a point, we must integrate the strains with respect to a fixed boundary condition. This integration can be conducted over many different paths. A solution is compatible if the displacement at all points is not a function of the path. Therefore, a displacement compatible solution involves the existence of a uniquely defined displacement field.

In the analysis of a structural system of discrete elements, all elements connected to a joint or node point must have the same absolute displacement. If the node displacements are given, all element deformations can be calculated from the basic equations of geometry. In a displacement-based finite element analysis, node displacement compatibility is satisfied. However, it is not necessary that the displacements along the sides of the elements be compatible if the element passes the "patch test."

A finite element passes the patch test "if a group (or patch) of elements, of arbitrary shape, is subjected to node displacements associated with constant strain; and the results of a finite element analysis of the patch of elements yield constant strain." In the case of plate bending elements, the application of a constant curvature displacement pattern at the nodes must produce constant curvature within a patch of elements. If an element does not pass the patch test, it may not converge to the exact solution. Also, in the case of a coarse mesh,

elements that do not pass the patch test may produce results with significant errors.

2.5 STRAIN DISPLACEMENT EQUATIONS

If the small displacement fields u_1, u_2 and u_3 are specified, assumed or calculated, the consistent strains can be calculated directly from the following well-known strain-displacement equations[2]:

$$\varepsilon_1 = \frac{\partial u_1}{\partial x_1} \tag{2.3a}$$

$$\varepsilon_2 = \frac{\partial u_2}{\partial x_2} \tag{2.3b}$$

$$\varepsilon_3 = \frac{\partial u_3}{\partial x_3} \tag{2.3c}$$

$$\gamma_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \tag{2.3d}$$

$$\gamma_{13} = \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \tag{2.3e}$$

$$\gamma_{23} = \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \tag{2.3f}$$

2.6 DEFINITION OF ROTATION

A unique rotation at a point in a real structure does not exist. A rotation of a horizontal line may be different from the rotation of a vertical line. However, in many theoretical books on continuum mechanics the following mathematical equations are used to define rotation of the three axes:

$$\theta_3 \equiv \frac{1}{2}\left[\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right] \tag{2.4a}$$

$$\theta_2 \equiv \frac{1}{2}\left[\frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}\right] \tag{2.4b}$$

$$\theta_1 \equiv \frac{1}{2}\left[\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right] \tag{2.4c}$$

It is of interest to note that this definition of rotation is the average rotation of two normal lines. It is important to recognize that these definitions are not the same as used in beam theory when shearing deformations are included. When beam sections are connected, the absolute rotation of the end sections must be equal.

2.7 EQUATIONS AT MATERIAL INTERFACES

One can clearly understand the fundamental equilibrium and compatibility requirements from an examination of the stresses and strains at the interface between two materials. A typical interface for a two-dimensional continuum is shown in Figure 2.1. By definition, the displacements at the interface are equal. Or, $u_s(s,n) = \overline{u}_s(s,n)$ and $u_n(s,n) = \overline{u}_n(s,n)$.

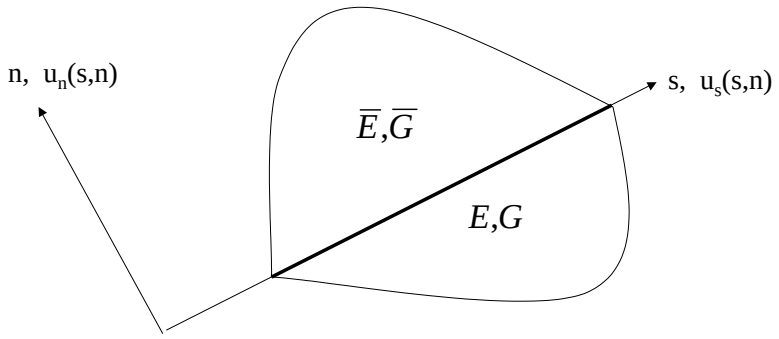

Figure 2.1 Material Interface Properties

Normal equilibrium at the interface requires that the normal stresses be equal. Or:

$$\sigma_n = \overline{\sigma}_n \tag{2.5a}$$

Also, the shear stresses at the interface are equal. Or:

$$\tau_{ns} = \bar{\tau}_{ns} \tag{2.5b}$$

Because displacement u_s and $\bar{u}_s$ must be equal and continuous at the interface:

$$\varepsilon_s = \bar{\varepsilon}_s \tag{2.5c}$$

Because the material properties that relate stress to strain are not equal for the two materials, it can be concluded that:

$$\sigma_s \neq \bar{\sigma}_s \tag{2.5d}$$

$$\varepsilon_n \neq \bar{\varepsilon}_n \tag{2.5e}$$

$$\gamma_{ns} \neq \bar{\gamma}_{ns} \tag{2.5f}$$

For a three-dimensional material interface on a s-t surface, it is apparent that the following 12 equilibrium and compatibility equations exist:

$$\sigma_n = \bar{\sigma}_n \qquad \varepsilon_n \neq \bar{\varepsilon}_n \tag{2.6a}$$

$$\sigma_s \neq \bar{\sigma}_s \qquad \varepsilon_s = \bar{\varepsilon}_s \tag{2.6b}$$

$$\sigma_t \neq \bar{\sigma}_t \qquad \varepsilon_t = \bar{\varepsilon}_t \tag{2.6c}$$

$$\tau_{ns} = \bar{\tau}_{ns} \qquad \gamma_{ns} \neq \bar{\gamma}_{ns} \tag{2.6d}$$

$$\tau_{nt} = \bar{\tau}_{nt} \qquad \gamma_{nt} \neq \bar{\gamma}_{nt} \tag{2.6e}$$

$$\tau_{st} \neq \bar{\tau}_{st} \qquad \gamma_{st} = \bar{\gamma}_{st} \tag{2.6f}$$

These 12 equations cannot be derived because they are fundamental physical laws of equilibrium and compatibility. It is important to note that if a stress is continuous, the corresponding strain, derivative of the displacement, is

discontinuous. Also, if a stress is discontinuous, the corresponding strain, derivative of the displacement, is continuous.

The continuity of displacements between elements and at material interfaces is defined as C_0 displacement fields. Elements with continuities of the derivatives of the displacements are defined by C_1 continuous elements. It is apparent that elements with C_1 displacement compatibility cannot be used at material interfaces. Therefore, the rotations, as defined by Equations 2.4 are not continuous at material interfaces.

2.8 INTERFACE EQUATIONS IN FINITE ELEMENT SYSTEMS

In the case of a finite element system in which the equilibrium and compatibility equations are satisfied only at node points along the interface, the fundamental equilibrium equations can be written as:

$$\sum F_n + \sum \bar{F}_n = 0 \tag{2.7a}$$

$$\sum F_s + \sum \bar{F}_s = 0 \tag{2.7b}$$

$$\sum F_t + \sum \bar{F}_t = 0 \tag{2.7c}$$

Each node on the interface between elements has a unique set of displacements; therefore, compatibility at the interface is satisfied at a finite number of points. As the finite element mesh is refined, the element stresses and strains approach the equilibrium and compatibility requirements given by Equations (2.6a) to (2.6f). Therefore, each element in the structure may have different material properties; and, special interface equations are required at material interfaces.

The discussion in this Chapter to this point applies to three-dimensional elastic solids only. In addition, it clearly indicates the difference between classical elasticity and the modern finite element method exactly satisfy equilibrium as the mesh is refined. Also, in my opinion, it is prove that displacement compatible finite element solutions will converge to the exact elasticity solution as the mesh is refined.

2.9 NODE ROTATIONS IN FINITE ELEMENT SYSTEMS

Gustave Kirchhoff (1824-1887) [3], in a paper on the theory of thin plates, introduced the following approximation: **under small deflections, each line which is initially perpendicular to the middle plane of the plate remains straight during bending and normal to the middle surface of the deflected plate.** In modern structural analysis the normal rotations of the normal line are the two unknown node rotations. However, if shearing deformations are included the plate, beam or shell element the average normal line rotation is not the same as the rotations of the middle surface of the plate.

The membrane formulation for the plate and shell elements, as presented in Chapters 9 and 10, introduces a normal node rotation in order to allow more flexibility in the connection of complex beam, plate and shell elements to model the three-dimensional behavior of complex structural systems. However, at the intersection of elements of different materials or thicknesses, great care must be taken to impose the appropriate interface continuity conditions. For example, Appendix K illustrates how to model the behavior of a horizontal floor slab with a vertical shear wall..

2.10 STATICALLY DETERMINATE STRUCTURES

The internal forces of some structures can be determined directly from the equations of equilibrium only. For example, the truss structure shown in Figure 2.2 will be analyzed to illustrate that the classical "method of joints" is nothing more than solving a set of equilibrium equations.

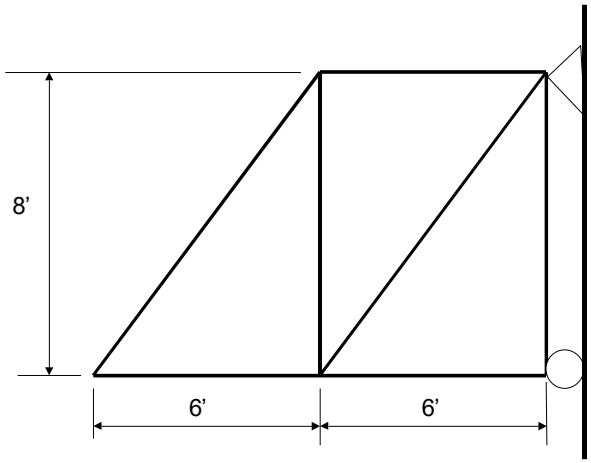

Figure 2.2 Simple Truss Structure

Positive external node loads and node displacements are shown in Figure 2.3. Member forces f_i and deformations d_i are positive in tension.

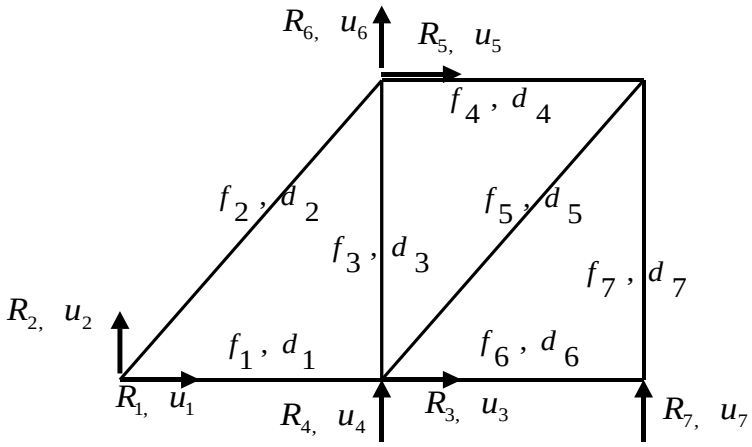

Figure 2.3 Definition of Positive Joint Forces and Node Displacements

Equating two external loads, R_j, at each joint to the sum of the internal member forces, f_i, (see Appendix B for details) yields the following seven equilibrium equations written as one matrix equation:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \end{bmatrix} = \begin{bmatrix} -1.0 & -0.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & -0.6 & 0 & 0 \\ 0 & 0 & -1.0 & 0 & -0.8 & -1.0 & 0 \\ 0 & 0.6 & 0 & -1.0 & 0 & 0 & 0 \\ 0 & 0.8 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{bmatrix} \quad (2.8)$$

Or, symbolically:

$$\mathbf{R} = \mathbf{Af} \quad (2.9)$$

where $\mathbf{A}$ is a load-force transformation matrix and is a function of the geometry of the structure only. For this ***statically determinate*** structure, we have seven unknown element forces and seven joint equilibrium equations; therefore, the above set of equations can be solved directly for any number of joint load conditions. If the structure had one additional diagonal member, there would be eight unknown member forces, and a direct solution would not be possible because the structure would be ***statically indeterminate***. The major purpose of this example is to express the well-known traditional method of analysis ("***method of joints"***) in matrix notation.

2.11 DISPLACEMENT TRANSFORMATION MATRIX

After the member forces have been calculated, there are many different traditional methods to calculate joint displacements. Again, to illustrate the use of matrix notation, the member deformations d_i will be expressed in terms of joint displacements u_j. Consider a typical truss element as shown in Figure 2.4.

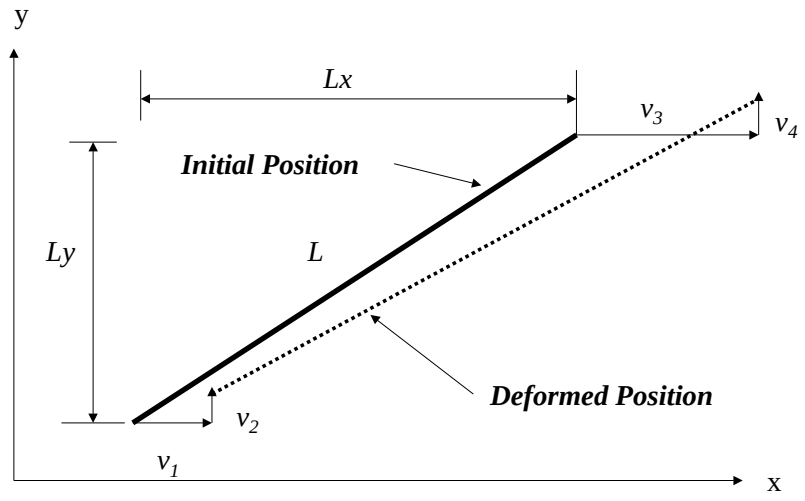

Figure 2.4 Typical Two-Dimension Truss Element

The axial deformation of the element can be expressed as the sum of the axial deformations resulting from the four displacements at the two ends of the element. The total axial deformation written in matrix form is:

$$d = \begin{bmatrix} -\frac{L_x}{L} & -\frac{L_y}{L} & \frac{L_x}{L} & \frac{L_y}{L} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{2.10}$$

Application of Equation (2.10) to all members of the truss shown in Figure 2.3 yields the following matrix equation:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \end{bmatrix} = \begin{bmatrix} -1.0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ -0.6 & -0.8 & 0 & 0 & 0.6 & 0.8 & 0 \\ 0 & 0 & 0 & -1.0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & -1.0 & 0 & 0 \\ 0 & 0 & -0.6 & -0.8 & 0 & 0 & 0 \\ 0 & 0 & -1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} \tag{2.11}$$

Or, symbolically:

$$\mathbf{d} = \mathbf{B}\,\mathbf{u} \tag{2.12}$$

The element deformation-displacement transformation matrix, **B,** is a function of the geometry of the structure. Of greater significance, however, is the fact that the matrix **B** is the transpose of the matrix **A** defined by the joint equilibrium Equation (2.8). Therefore, given the element deformations within this statically determinate truss structure, we can solve Equation (2.11) for the joint displacements.

2.12 ELEMENT STIFFNESS AND FLEXIBILITY MATRICES

The forces in the elements can be expressed in terms of the deformations in the elements using the following matrix equations:

$$\mathbf{f} = \mathbf{k}\,\mathbf{d} \qquad \text{or,} \qquad \mathbf{d} = \mathbf{k}^{-1}\,\mathbf{f} \tag{2.13}$$

The element stiffness matrix **k** is diagonal for this truss structure, where the diagonal terms are $k_{ii} = \dfrac{A_i E_i}{L_i}$ and all other terms are zero. The element flexibility matrix is the inverse of the stiffness matrix, where the diagonal terms are $\dfrac{L_i}{A_i E_i}$. It is important to note that the element stiffness and flexibility matrices are only a function of the mechanical properties of the elements.

2.13 SOLUTION OF STATICALLY DETERMINATE SYSTEM

The three fundamental equations of structural analysis for this simple truss structure are equilibrium, Equation (2.8); compatibility, Equation (2.11); and force-deformation, Equation (2.13). For each load condition R, the solution steps can be summarized as follows:

1. Calculate the element forces from Equation (2.8).

2. Calculate element deformations from Equation (2.13).

3. Solve for joint displacements using Equation (2.11).

All traditional methods of structural analysis use these basic equations. However, before the availability of inexpensive digital computers that can solve over 100 equations in less than one second, many special techniques were developed to minimize the number of hand calculations. Therefore, at this point in time, there is little value to summarize those methods in this book on the static and dynamic analysis of structures.

2.14 GENERAL SOLUTION OF STRUCTURAL SYSTEMS

In structural analysis using digital computers, the same equations used in classical structural analysis are applied. The starting point is always joint equilibrium. Or, $\mathbf{R} = \mathbf{A}\,\mathbf{f}$. From the element force-deformation equation, $\mathbf{f} = \mathbf{k}\,\mathbf{d}$, the joint equilibrium equation can be written as $\mathbf{R} = \mathbf{A}\,\mathbf{k}\,\mathbf{d}$. From the compatibility equation, $\mathbf{d} = \mathbf{B}\,\mathbf{u}$, joint equilibrium can be written in terms of joint displacements as $\mathbf{R} = \mathbf{A}\,\mathbf{k}\,\mathbf{B}\,\mathbf{u}$. Therefore, the general joint equilibrium can be written as:

$$\mathbf{R} = \mathbf{K}\,\mathbf{u} \tag{2.14}$$

The global stiffness matrix **K** is given by one of the following matrix equations:

$$\mathbf{K} = \mathbf{A}\,\mathbf{k}\,\mathbf{B} \quad \text{or} \quad \mathbf{K} = \mathbf{A}\,\mathbf{k}\,\mathbf{A}^{\mathbf{T}} \quad \text{or} \quad \mathbf{K} = \mathbf{B}^{\mathbf{T}}\,\mathbf{k}\,\mathbf{B} \tag{2.15}$$

It is of interest to note that the equations of equilibrium or the equations of compatibility can be used to calculate the global stiffness matrix **K**.

The standard approach is to solve Equation (2.14) for the joint displacements and then calculate the member forces from:

$$\mathbf{f} = \mathbf{k}\,\mathbf{B}\,\mathbf{u} \quad \text{or} \quad \mathbf{f} = \mathbf{k}\,\mathbf{A}^{\mathbf{T}}\,\mathbf{u} \tag{2.16}$$

It should be noted that within a computer program, the sparse matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{k}$ and $\mathbf{K}$ are never formed because of their large storage requirements. The symmetric global stiffness matrix $\mathbf{K}$ is formed and solved in condensed form.

2.15 SUMMARY

Internal member forces and stresses must be in equilibrium with the applied loads and displacements. All real structures satisfy this fundamental law of physics. Hence, our computer models must satisfy the same law.

At material interfaces, all stresses and strains are not continuous. Computer programs that average node stresses at material interfaces produce plot stress contours that are continuous; however, the results will not converge and significant errors can be introduced by this approximation.

Compatibility conditions, which require that all elements attached to a rigid joint have the same displacement, are fundamental requirements in structural analysis and can be physically understood. Satisfying displacement compatibility involves the use of simple equations of geometry. However, the compatibility equations have many forms, and most engineering students and many practicing engineers can have difficulty in understanding the displacement compatibility requirement. Some of the reasons we have difficulty in the enforcement of the compatibility equations are the following:

1. The displacements that exist in most linear structural systems are small compared to the dimensions of the structure. Therefore, deflected shape drawing must be grossly exaggerated to write equations of geometry.

2. For structural systems that are statically determinate, the internal member forces and stresses can be calculated exactly without the use of the compatibility equations.

3. Many popular (approximate) methods of analysis exist that do not satisfy the displacement compatibility equations. For example, for rectangular frames, both the cantilever and portal methods of analysis assume the inflection points to exist at a predetermined location within the beams or columns; therefore, the displacement compatibility equations are not satisfied.

4. Many materials, such as soils and fluids, do not satisfy the compatibility equations. Also, locked in construction stresses, creep and slippage within joints are real violations of displacement compatibility. Therefore, approximate methods that satisfy statics may produce more realistic results for the purpose of design.

5. In addition, engineering students are not normally required to take a course in geometry; whereas, all students take a course in statics. Hence, there has not been an emphasis on the application of the equations of geometry.

The relaxation of the displacement compatibility requirement has been justified for hand calculation to minimize computational time. Also, if one must make a choice between satisfying the equations of statics or the equations of geometry, in general, we should satisfy the equations of statics for the reasons previously stated.

However, because of the existence of inexpensive powerful computers and efficient modern computer programs, it is not necessary to approximate the compatibility requirements. For many structures, such approximations can produce significant errors in the force distribution in the structure in addition to incorrect displacements.

2.16 REFERENCES

1. Cook, R. D., D. S. Malkus and M. E. Plesha. 1989. *Concepts and Applications of Finite Element Analysis,* Third Edition. John Wiley & Sons, Inc. ISBN 0-471-84788-7.

2. Boresi, A. P. 1985. *Advanced Mechanics of Materials.* John Wiley & Sons, Inc. ISBN 0-471-88392-1.

3. Timoshenko, Stephen P. *History of the Strength of Materials, Dover Publication, Inc. 1983, Originally published by McGraw-Hill, 1953, ISBN 0-486-61187-6.*

APPENDIX H

SPEED OF COMPUTER SYSTEMS

The Current Speed of a $2,000 Personal Computer is Faster than the $10,000,000 Cray Computer of 1975

INTRODUCTION

The calculation of element stiffness matrices, solution of equations and evaluation of mode shapes and frequencies are all computationally intensive. Furthermore, it is necessary to use double-precision floating-point arithmetic to avoid numerical errors. Therefore, all numbers must occupy 64 bits of computer storage. The author started developing structural analysis and design programs on the IBM-701 in 1957 and since that time has been exposed to a large number of different computer systems. In this appendix the approximate double-precision floating-point performances of some of those computer systems are summarized. Because different FORTRAN compilers and operating systems were used, the speeds presented can only be considered accurate to within 50 percent.

DEFINITION OF ONE NUMERICAL OPERATION

For the purpose of comparing floating-point speeds, the evaluation of the following equation is defined as one operation:

```
A = B + C * D      Definition of one numerical operation
```

Using double precision arithmetic, the definition involves the sum of one multiplication, one addition, extracting three numbers from high-speed storage, and transferring the results to storage. In most cases, this type of operation is within the inner DO LOOP for the solution of linear equations and the evaluation of mode shapes and frequencies.

SPEED OF DIFFERENT COMPUTER SYSTEMS

Table H.1 indicates the speed of different computers used by the author.

Table H.1 Floating-Point Speeds of Computer Systems

Year	Computer or CPU	Operations Per Second	Relative Speed
1963	CDC-6400	50,000	1
1967	CDC-6600	100,000	2
1974	CRAY-1	3,000,000	60
1980	VAX-780	60,000	1.2
1981	IBM-3090	20,000,000	400

Year	Computer or CPU	Operations Per Second	Relative Speed
1981	CRAY-XMP	40,000,000	800
1990	DEC-5000	3,500,000	70
1994	Pentium-90	3,500,000	70
1995	Pentium-133	5,200,000	104
1995	DEC-5000 upgrade	14,000,000	280
1998	Pentium II - 333	37,500,000	750
1999	Pentium III - 450	69,000,000	1,380

If one considers the initial cost and maintenance of the various computer systems, it is apparent that the overall cost of engineering calculations has reduced significantly during the past several years. The most cost effective computer system at the present time is the INTEL Pentium III type of personal computer system. At the present time,

a very powerful personal computer system that is 25 times faster than the first CRAY computer, the fastest computer made in 1974, can be purchased for approximately $1,500.

SPEED OF PERSONAL COMPUTER SYSTEMS

Many engineers do not realize the computational power of the present day inexpensive personal computer. Table H.2 indicates the increased speed of personal computers that has occurred during the past 18 years.

Table H.2 Floating-Point Speeds of Personal Computer Systems

YEAR	INTEL CPU	Speed MHz	Operations Per Second	Relative Speed	COST
1980	8080	4	200	1	$6,000
1984	8087	10	13,000	65	$2,5

					0 0
19 88	8038 7	2 0	93,0 00	4 6 5	$ 8 , 0 0 0
19 91	8048 6	3 3	605, 000	3, 0 2 5	$ 1 0 , 0 0 0
19 94	8048 6	6 6	1,21 0,00 0	6, 0 5 0	$ 5 , 0 0 0
19 95	Penti um	9 0	4,00 0,00 0	2 6, 0 0 0	$ 5 , 0 0 0
19 96	Penti um	2 3 3	10,3 00,0 00	5 2, 0 0 0	$ 4 , 0 0 0

1997	Pentium II	233	11,500,000	58,000	$3,000
1998	Pentium II	333	37,500,000	198,000	$2,500
1999	Pentium III	450	69,000,000	345,000	$1,500

One notes that the floating-point speed of the Pentium III is significantly different from the Pentium II chip. The increase in clock speed, from 333 to 450 MHz, does not account for the increase in speed.

PAGING OPERATING SYSTEMS

The above computer speeds assume all numbers are in high-speed memory. For the analysis of large structural systems, it is not possible to store all information within high-speed storage. If data needs to be obtained from low-speed disk storage, the effective speed of a computer can be reduced significantly. Within the SAP and ETABS programs, the transfer of data to and from disk storage is conducted in large blocks to minimize disk access time. That programming philosophy was used before introduction of the paging option used in the modern Windows operating systems.

In a paging operating system, if the data requested is not stored in high-speed memory, the computer automatically reads the data from disk storage in relatively small blocks of information. Therefore, the modern programmer need not be concerned with data management. However, there is a danger in the application of this approach. The classical example that illustrates the problem with paging is adding two large matrices together. The FORTRAN statement can be one of the following forms:

```
    DO 100 J=1,NCOL DO 100 I=1,NROW

    DO 100 I=1,NROW DO 100 J=1,NCOL

100 A(I,J)=B(I,J)+C(I,J)  100A(I,J)=B(I,J)+C(I,J)
```

Because all arrays are stored row-wise, the data will be paged to and from disk storage in the same order as needed by the program statements on the left. However, if the program statements on the right are used, the computer may be required to read and write blocks of data to the disk for each term in the matrix. Hence, the computer time required for this simple operation can be very large if paging is automatically used.

SUMMARY

Personal computers will continue to increase in speed and decrease in price. It is the opinion of many experts in the field that the only way significant increases in speed will occur is by the addition of multi-processors to personal computer systems. The NT operating system supports the use of multi-processors. However, the ***free*** LINUX operating system has proven faster for many functions.