19.

LINEAR VISCOUS DAMPING

Linear Viscous Damping
Is a Property of the Computational Model
And is not a Property of a Real Structure

19.1 INTRODUCTION

{ XE "Viscous Damping" }In structural engineering, viscous velocity-dependent damping is very difficult to visualize for most real structural systems. Only a small number of structures have a finite number of damping elements where real viscous dynamic properties can be measured. In most cases modal damping ratios are used in the computer model to approximate unknown nonlinear energy dissipation within the structure.

{ XE "Damping:Rayleigh" }{ XE "Rayleigh Damping" }Another form of damping, referred to as Rayleigh damping, is often used in the mathematical model for the simulation of the dynamic response of a structure; Rayleigh damping is proportional to the stiffness and mass of the structure. Both modal and Rayleigh damping are used to avoid the need to form a damping matrix based on the physical properties of the real structure.

In recent years, the addition of energy dissipation devices to the structure has forced the structural engineer to treat the energy dissipation in a more exact manner. However, the purpose of this chapter is to discuss the limitations of modal and Rayleigh damping.

19.2 ENERGY DISSIPATION IN REAL STRUCTURES

It is possible to estimate an “effective or approximate” viscous damping ratio directly from laboratory or field tests of structures. One method is to apply a static displacement by attaching a cable to the structure and then suddenly removing the load by cutting the cable. If the structure can be approximated by a single degree of freedom, the displacement response will be of the form shown in Figure 19.1. For multi degree of freedom structural systems, the response will contain more modes and the analysis method required to predict the damping ratios will be more complex.

It should be noted that the decay of the typical displacement response only indicates that energy dissipation is taking place. The cause of the energy dissipation may be from many different effects such as material damping, joint friction and radiation damping at the supports. However, if it is assumed that all energy dissipation is the result of linear viscous damping, the free vibration response is given by the following equation:

$$u(t) = u(0)e^{-\xi\omega t}\cos(\omega_D t) \tag{19.1}$$

where : $\omega_D = \omega\sqrt{1-\xi^2}$

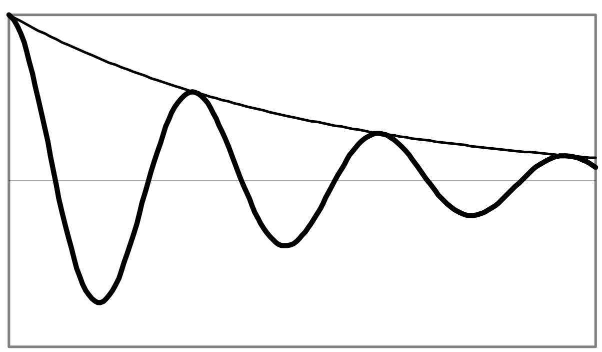

Figure 19.1 Free Vibration Test of Real Structures, Response vs. Time

Equation (19.1) can be evaluated at any two maximum points "m cycles" apart and the following two equations are produced:

$$u(2\pi n) = u_n = u(0)e^{-\xi\omega 2\pi n/\ \omega_D} \tag{19.2}$$

$$u(2\pi(n+m)) = u_{n+m} = u(0)e^{-\xi\omega 2\pi(n+m)/\ \omega_D} \tag{19.3}$$

{ XE "Algorithms for:Evaluation of Damping" }The ratio of these two equations is:

$$\frac{u_{n+m}}{u_n} = e^{-\frac{2\pi m\xi}{\sqrt{1-\xi^2}}} = r_m \tag{19.4}$$

{ XE "Damping:Decay Ratio" }Taking the natural logarithm of this decay ratio, r_m, and rewriting produces the following equation:

$$\xi = \frac{-\ln(r_m)}{2\pi m}\sqrt{1-\xi^2} \tag{19.5a}$$

This equation can be written in iterative form as:

$$\xi_{(i)} = \xi_0\sqrt{1-\xi_{(i-1)}^2} \tag{19.5b}$$

If the decay ratio equals 0.730 between two adjacent maximums, three iterations yield the following damping ratio to three significant figures:

$$\xi \approx 0.0501 \approx 0.0500 = 0.0500$$

{ XE "Damping:Classical Damping" }The damping value obtained by this approach is often referred to as ***effective damping***. Linear modal damping is also referred to as ***classical damping***. However, it must be remembered that it is an approximate value and is based on many assumptions.

Another type of energy dissipation that exists in real structures is radiation damping at the supports of the structure. The vibration of the structure strains the foundation material near the supports and causes stress waves to radiate into the infinite foundation. This can be significant if the foundation material is soft relative to the stiffness of the structure. The presence of a spring, damper and mass at each support often approximates this type of damping.

19.3 PHYSICAL INTERPRETATION OF VISCOUS DAMPING

The strain energy stored within a structure is proportional to the displacement squared. Hence, the amount of energy that is dissipated during each cycle of free vibration can be calculated for various damping ratios, as summarized in Table 19.1. In addition, Table 19.1 shows the number of cycles required to reduce the initial response by a factor of 10.

{ XE "Damping:Energy Loss Per Cycle" }**Table 19.1 Energy Loss Per Cycle for Different Damping Ratios**

Damping Ratio Percentage	Decay Ratio $r = e^{-\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$	Percentage Energy Loss Per Cycle $100\,(1-r^2)$	Number of Cycles to Damp Response by a Factor of 10 $n = \ln(0.10)/\ \ln(r)$
1	0.939	11.8	36.6
5	0.730	46.7	7.3
10	0.532	71.7	3.6
20	0.278	92.3	1.8
30	0.139	98.1	1.2

A 5 percent damping ratio indicates that 46.7 percent of the strain energy is dissipated during each cycle. If the period associated with the mode is 0.05 seconds, the energy is reduced by a factor of 10 in 0.365 second. Therefore, a 5 percent modal damping ratio produces a significant effect on the results of a dynamic response analysis.

Field testing of real structures subjected to small displacements indicates typical damping ratios are less than 2 percent. Also, for most structures, the damping is not linear and is not proportional to velocity. Consequently, values of modal damping over 5 percent are difficult to justify. However, it is often common practice for structural engineers to use values over 10 percent.

19.4 MODAL DAMPING VIOLATES DYNAMIC EQUILIBRIUM

{ XE "Damping:Experimental Evaluation" }{ XE "Damping:Equilibrium Violation" }For multi degree of freedom systems, the use of modal damping violates dynamic equilibrium and the fundamental laws of physics. For example, it is possible to calculate the reactions as a function of time at the base of a structure using the following two methods:

First, the inertia forces at each mass point can be calculated in a specific direction by multiplying the absolute acceleration in that direction times the mass at the point. In the case of earthquake loading, the sum of all these forces must be equal to the sum of the base reaction forces in that direction because no other forces act on the structure.

Second, the member forces at the ends of all members attached to reaction points can be calculated as a function of time. The sum of the components of the member forces in the direction of the load is the base reaction force experienced by the structure.

In the case of zero modal damping, those reaction forces, as a function of time, are identical. However, for nonzero modal damping, those reaction forces are significantly different. These differences indicate that linear modal damping introduces external loads that are acting on the structure above the base and are physically impossible. This is clearly an area where the standard "state-of-the-art" assumption of modal damping needs to be re-examined and an alternative approach developed.

Energy dissipation exists in real structures. However, it must be in the form of equal and opposite forces between points within the structure. Therefore, a viscous damper, or any other type of energy dissipating device, connected between two points within the structure is physically possible and will not cause an error in the reaction forces. There must be zero base shear for all internal energy dissipation forces.

19.5 NUMERICAL EXAMPLE

To illustrate the errors involved in the use of modal damping, a simple seven-story building was subjected to a typical earthquake motion. Table 19.2 indicates the values of base shear calculated from the external inertia forces, which satisfy dynamic equilibrium, and the base shear calculated from the exact summation of the shears at the base of the three columns.

It is of interest to note that the maximum values of base shear calculated from two different methods are significantly different for the same computer run. The only logical explanation is that the external damping forces exist only in the mathematical model of the structure. Because this is physically impossible, the use of standard modal damping can produce a small error in the analysis.

Table 19.2 Comparison of Base Shear for Seven-Story Building

Damping Percentage	Dynamic Equilibrium Base Shear (kips)	Sum of Column Shears (kips)	Error Percentage
0	370.7 @ 5.355 Sec.	370.7 @ 5.355 Sec.	0.0
2	314.7 @ 4.690 Sec	318.6 @ 4.695 Sec	+1.2
5	253.7 @ 4.675 Sec	259.6 @ 4.690 Sec	+2.3
10	214.9 @ 3.745 Sec	195.4 @ 4.035 Sec	-9.1
20	182.3 @ 3.055 Sec	148.7 @ 3.365 Sec	-18.4

It is of interest to note that the use of only 5 percent damping reduces the base shear from 371 kips to 254 kips for this example. Because the measurement of damping in most real structures has been found to be less than 2 percent, the selection of 5 percent reduces the results significantly.

19.6 STIFFNESS AND MASS PROPORTIONAL DAMPING

A very common type of damping used in the nonlinear incremental analysis of structures is to assume that the damping matrix is proportional to the mass and stiffness matrices. Or:

$$\mathbf{C} = \eta\mathbf{M} + \delta\mathbf{K} \tag{19.6}$$

{ XE "Damping:Rayleigh" }{ XE "Rayleigh Damping" }This type of damping is normally referred to as Rayleigh damping. In mode superposition analysis, the damping matrix must have the following properties in order for the modal equations to be uncoupled:

$$\begin{aligned} 2\omega_n \zeta_n &= \phi_n^T \mathbf{C} \phi_n = \eta \phi_n^T \mathbf{M} \phi_n + \delta \phi_n^T \mathbf{K} \phi_n \\ 0 &= \phi_n^T \mathbf{C} \phi_m \qquad n \neq m \end{aligned} \tag{19.7}$$

Because of the orthogonality properties of the mass and stiffness matrices, this equation can be rewritten as:

$$2\omega_n \zeta_n = \eta + \delta \omega_n^2 \qquad \text{or} \qquad \zeta_n = \frac{1}{2\omega_n}\eta + \frac{\omega_n}{2}\delta \tag{19.8}$$

It is apparent that modal damping can be specified exactly at only two frequencies, i and j , to solve for η and δ in the following equation:

$$\begin{bmatrix} \xi_i \\ \xi_j \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{bmatrix} \eta \\ \delta \end{bmatrix} \quad \text{For } \xi_i = \xi_j = \xi \quad \begin{Bmatrix} \delta = \dfrac{2\xi}{\omega_i + \omega_j} \\ \eta = \omega_i \omega_j \delta \end{Bmatrix} \tag{19.9}$$

{ XE "Damping:Stiffness Proportional" }For the typical case, the damping is set to be equal at the two frequencies; therefore $\xi_i = \xi_j = \xi$ and the proportionality factors are calculated from:

$$\delta = \frac{2\xi}{\omega_i + \omega_j} \quad \text{and} \quad \eta = \omega_i \omega_j \delta \tag{19.10}$$

{ XE "Damping:Mass Proportional" }The assumption of mass proportional damping implies the existence of external supported dampers that are physically impossible for a base supported structure. The use of stiffness proportional damping has the effect of increasing the damping in the higher modes of the structure for which there is no physical justification. This form of damping can result in significant errors for impact- type problems and earthquake displacement input at the base of a structure. Therefore, the use of Rayleigh-type damping is difficult to justify for most structures. However, it continues to be used within many computer programs to obtain numerical results using large time integration steps.

19.7 CALCULATION OF ORTHOGONAL DAMPING MATRICES

{ XE "Orthogonal Damping Matrices" }In Chapter 13, the classical damping matrix was assumed to satisfy the following orthogonality relationship:

$$\Phi^{\mathrm{T}}\mathbf{C}\,\Phi=\mathbf{d} \quad \text{where } \mathrm{d}_{\mathrm{nn}}=2\xi_n\omega_n \text{ and } d_{nm}=0 \quad \text{for } n\neq m \tag{19.11}$$

In addition, the mode shapes are normalized so that $\Phi^{\mathrm{T}}\mathbf{M}\Phi=\mathbf{I}$. The following matrix can be defined:

$$\overline{\Phi}=\Phi\mathbf{M} \quad \text{and} \quad \overline{\Phi}^{\mathrm{T}}=\mathbf{M}\Phi^{T} \tag{19.12}$$

Hence, if Equation 19.11 is pre-multiplied by $\overline{\Phi}$ and post-multiplied by $\overline{\Phi}^{\mathrm{T}}$, the following damping matrix is obtained:

$$\mathbf{C}=\overline{\Phi}\mathbf{d}\overline{\Phi}^{T}=\sum_{n=1}^{N}C_n \tag{19.13}$$

Therefore, a classical damping matrix can be calculated for each mode that has a specified amount of damping in that mode and zero damping in all other modes:

$$\mathbf{C}_n=2\xi_n\omega_n\,\mathbf{M}\phi_n\phi_n^T\mathbf{M} \tag{19.14}$$

It must be noted that this modal damping matrix is a mathematical definition and that it is physically impossible for such damping properties to exist in a real multi degree of freedom structure.

The total damping matrix for all modes can be written as:

$$\mathbf{C}=\sum_{n=1}^{N}\mathbf{C}_n=\sum_{\mathrm{n=1}}^{\mathrm{N}}2\xi_n\omega_n\,\mathbf{M}\phi_n\phi_n^T\mathbf{M} \tag{19.15}$$

It is apparent that given the mode shapes, a full damping matrix can be constructed from this mathematical equation. However, the resulting damping matrix may require that external dampers and negative damping elements be connected between nodes of the computer model.

The only reason to form such a damping matrix is to compare the results of a step-by-step integration solution with a mode superposition solution. A numerical example is given in reference [1].

19.8 STRUCTURES WITH NON-CLASSICAL DAMPING

{ XE "Damping:Nonlinear" }It is possible to model structural systems with linear viscous dampers at arbitrary locations within a structural system. The exact solution involves the calculation of complex eigenvalues and eigenvectors and a large amount of computational effort. Because the basic nature of energy dissipation is not clearly defined in real structures and viscous damping is often used to approximate nonlinear behavior, this increase in computational effort is not justified given that we are not solving the real problem. A more efficient method to solve this problem is to move the damping force to the right-hand side of the dynamic equilibrium equation and solve the problem as a nonlinear problem using the FNA method. Also, nonlinear viscous damping can easily be considered by this new computational method.

19.9 NONLINEAR ENERGY DISSIPATION

Most physical energy dissipation in real structures is in phase with the displacements and is a nonlinear function of the magnitude of the displacements. Nevertheless, it is common practice to approximate the nonlinear behavior with an “equivalent linear damping” and not conduct a nonlinear analysis. The major reason for this approximation is that all linear programs for mode superposition or response spectrum analysis can consider linear viscous damping in an exact mathematical manner. This approximation is no longer necessary if the structural engineer can identify where and how the energy is dissipated within the structural system. The FNA method provides an alternative to the use of equivalent linear viscous damping.

{ XE "Gap-Crush Element" }Base isolators are one of the most common types of predefined nonlinear elements used in earthquake resistant designs. Mechanical dampers, friction devices and plastic hinges are other types of common nonlinear elements. In addition, gap elements are required to model contact between structural components and uplifting of structures. A special type of gap element, with the ability to crush and dissipate energy, is useful to model concrete and soil types of materials. Cables that can take tension only and dissipate energy in yielding are necessary to capture the behavior of many bridge type structures. However, when a nonlinear analysis is conducted where energy is dissipated within the nonlinear devices, one cannot justify adding an additional 5 percent of linear modal damping

19.10 SUMMARY

The use of linear modal damping as a percentage of critical damping has been used to approximate the nonlinear behavior of structures. The energy dissipation in real structures is far more complicated and tends to be proportional to displacements rather than proportional to the velocity. The use of approximate "equivalent viscous damping" has little theoretical or experimental justification and produces a mathematical model that violates dynamic equilibrium.

One can mathematically create damping matrices to have different damping in each mode. In addition, one can use stiffness and mass proportional damping matrices. To justify these convenient mathematical assumptions, field experimental work must be conducted.

It is now possible to accurately simulate, using the FNA method, the behavior of structures with a finite number of discrete energy dissipation devices installed. The experimentally determined properties of the devices can be directly incorporated into the computer model.

Please note, no reference to **complex mode shapes** has been made in this Chapter. Complex mode shapes are a property of the mathematical equations that are used to **approximately** model the dynamic behavior of real structures. Complex mode shapes do not exist in real structures. What you can measure in a real structure, when it is subjected to dynamic loads, is a series of decaying shapes that vary with time. The decaying of the amplitude of the shapes is due to energy dissipation caused by many different physical properties of the structure such as friction and nonlinear behavior.

Complex mode shapes are only possible if the assumption of **linear viscous damping** is made. However, no one has ever created a device where the applied force is directly proportional to the measured velocity for all frequencies. Therefore, we can eliminate the requirement that it is necessary to learn complex variable analysis in order to perform accurate dynamic analysis of real structures***.(12/10/13)***

19.11 REFERENCES

1. Wilson, E., and J. Penzien. 1972. "Evaluation of Orthogonal Matrices," *International Journal for Numerical Methods in Engineering*. Vol. 4. pp. 5-10.