

APPENDIX E

TRANSFORMATION OF MATERIAL PROPERTIES

Many of the New Materials used in Structural Engineering Have Orthotropic Material Properties

E.1 INTRODUCTION

{ XE "Material Property Transformation" } Orthotropic material properties are defined in a local 1-2-3 coordinate system and are defined by the following equation:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{21} \\ \gamma_{31} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_2} & -\frac{\nu_{13}}{E_3} & -\frac{\nu_{14}}{E_4} & -\frac{\nu_{15}}{E_5} & -\frac{\nu_{16}}{E_6} \\ \frac{\nu_{21}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_3} & -\frac{\nu_{24}}{E_4} & -\frac{\nu_{25}}{E_5} & -\frac{\nu_{26}}{E_6} \\ \frac{\nu_{31}}{E_1} & \frac{\nu_{32}}{E_2} & \frac{1}{E_3} & -\frac{\nu_{34}}{E_4} & -\frac{\nu_{35}}{E_5} & -\frac{\nu_{36}}{E_6} \\ -\frac{\nu_{41}}{E_1} & -\frac{\nu_{42}}{E_2} & -\frac{\nu_{43}}{E_3} & \frac{1}{E_4} & -\frac{\nu_{45}}{E_5} & -\frac{\nu_{46}}{E_6} \\ -\frac{\nu_{51}}{E_1} & -\frac{\nu_{52}}{E_2} & -\frac{\nu_{53}}{E_3} & -\frac{\nu_{54}}{E_4} & \frac{1}{E_5} & -\frac{\nu_{56}}{E_6} \\ -\frac{\nu_{61}}{E_1} & -\frac{\nu_{62}}{E_2} & -\frac{\nu_{63}}{E_3} & -\frac{\nu_{64}}{E_4} & -\frac{\nu_{65}}{E_5} & \frac{1}{E_6} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{21} \\ \tau_{31} \\ \tau_{23} \end{bmatrix} + \Delta T \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{23} \end{bmatrix} \quad (\text{E.1})$$

Or in matrix notation:

$$\mathbf{d} = \mathbf{Cf} + \Delta T \mathbf{a} \quad (\text{E.2})$$

However, it is necessary to write equilibrium equations and other equations in a common "global" x-y-z coordinate system. Therefore, it is necessary for Equation (E.2) to be converted, or rotated, to the x-y-z system.

The classical equation for three-dimensional stress transformation can be written, by considering the equilibrium of a three-dimensional element, as the following matrix equation:

$$\begin{bmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{21} & \sigma_2 & \tau_{23} \\ \tau_{31} & \tau_{32} & \sigma_3 \end{bmatrix} = \begin{bmatrix} V_{x1} & V_{y1} & V_{z1} \\ V_{x2} & V_{y2} & V_{z2} \\ V_{x3} & V_{y3} & V_{z3} \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \begin{bmatrix} V_{x1} & V_{x2} & V_{x3} \\ V_{y1} & V_{y2} & V_{y3} \\ V_{z1} & V_{z2} & V_{z3} \end{bmatrix} \quad (\text{E.3})$$

where V_{xi} , V_{yi} , and V_{zi} are the direction cosines of axis "i" with respect to the global x-y-z system. Equation (E.3) can be expanded to nine scalar equations. However, because of equilibrium, only six independent stresses exist in each system. Therefore, the 6 stresses in the local system can be written in terms of 6 global stresses in the following form:

$$\sigma = \mathbf{a} \sigma_g \quad (\text{E.4})$$

where "a" is a 6 by 6 stress transformation matrix that must be numerically formed for each different element within a structural system. One approach would be to form analytical expressions, in terms of the products of the direction cosines, for each of the 36 terms in the matrix. An alternative to this traditional algebraic approach is to numerically evaluate, within the computer program, the 6 by 6 matrix directly from the 3 by 3 direction cosine matrix. This simple approach is best illustrated by the FORTRAN subroutine given in Table E.1. One notes that a 3 by 3 integer array "IJ" is used to map the 3 by 3 stress to a 6 by 1 column matrix.

Table E.1 Formation of the "a" Matrix

```

SUBROUTINE CALA(A,V)
DIMENSION A(6,6),V(4,3),IJ(3,3)
DATA IJ/1,4,5, 4,2,6, 5,6,3/
C----- ZERO 6 by 6 STRESS TRANSFORMATION MATRIX -----
DO 100 I=1,6
DO 100 J=1,6
100 A(I,J) = 0.0
C----- FORM "A" ARRAY -----
DO 400 II=1,3
DO 400 JJ=II,3
I = IJ(II,JJ)
DO 300 K=1,3
DO 300 L=1,3
J = IJ(K,L)
300 A(I,J) = A(I,J) + V(K,II)*V(L,JJ)
400 CONTINUE
C----- MATRIX FORMED -----
RETURN
END

```

Also, the classical equations for strain transformation can be written as:

$$\varepsilon_g = \mathbf{a}^T \varepsilon \quad (\text{E.5})$$

Equation (E.1) can now be written in the global x-y-z system as:

$$\varepsilon_g = \mathbf{C}_g \varepsilon + \varepsilon_{og} \quad (\text{E.6})$$

where:

$$\mathbf{C}_g = \mathbf{a}^T \mathbf{C} \mathbf{a} \quad (\text{E.7})$$

$$\varepsilon_{og} = \Delta T \mathbf{a}^T \alpha \quad (\text{E.8})$$

Because each member of a complex structural system may have different orthotropic material properties, the matrix multiplication required to calculate Equations (E.7) and (E.8) are numerically evaluated within the computer program before inversion of \mathbf{C}_g .

E.2 SUMMARY

Many material properties are orthotropic. In the past the structural engineer has often neglected to use those properties because of the increase in hand

computational requirements. However, material properties can be easily incorporated into modern computer programs without a significant increase in computational time. The necessary equations to transform those local properties within each element to a common global reference system have been presented in this appendix.